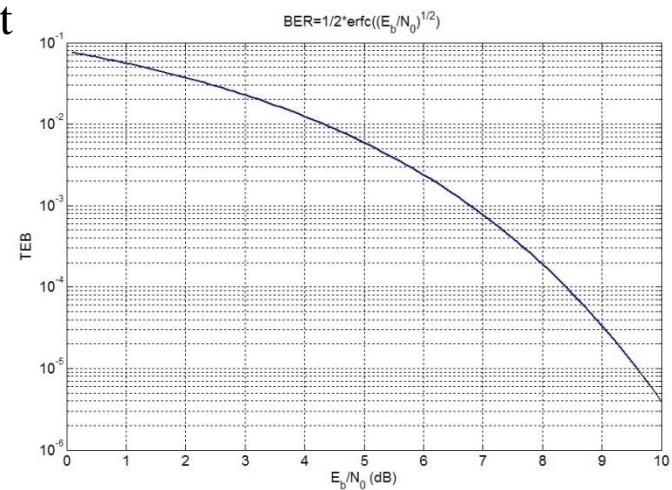
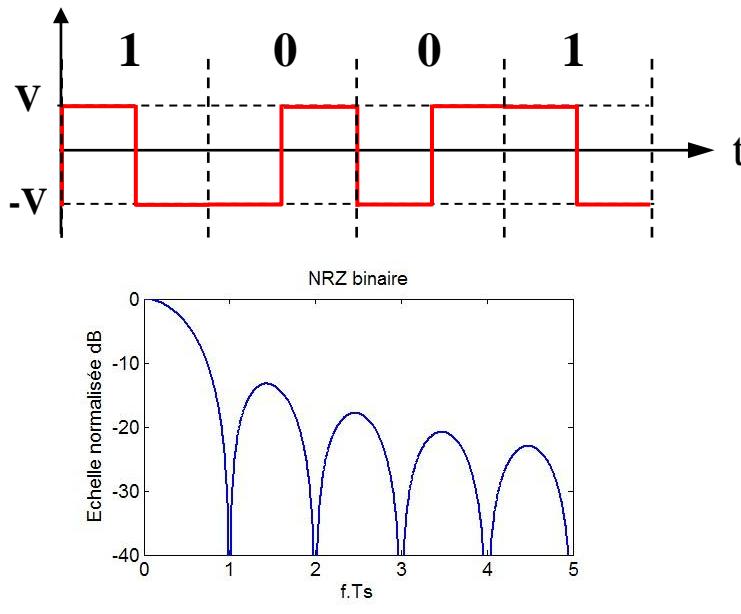


Signal processing Digital Communications

ERASMUS MUNDUS Monabiphot Master



Eric Vourc'h

eric.vourch@satie.ens-cachan.fr

Ecole Normale Supérieure de Cachan / SATIE laboratory

Course outline

- ✓ Introduction
- ✓ Baseband transmissions
 - ✓ Source coding
 - ✓ BER optimization
 - Channel sounding
 - Channel coding
 - Interleaving
 - ✓ In line coding
- ✓ Transmission over a carrier frequency

- Time variations

- Spectrum

- ...

- Baseband transmissions

- Transmissions over a carrier frequency

- ...

- The characteristics of the transmission channel :

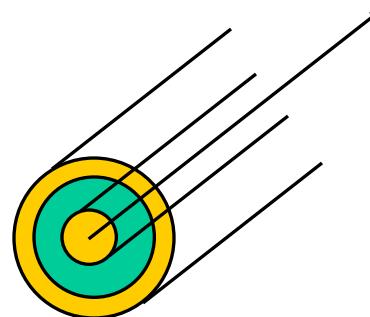
The choice of the transmission technique (baseband or carrier frequency transmission) mainly depends on the characteristics of the transmission channel (medium)

Transmission channel : medium or waveguide where the signal are transmitted (cable, waveguide, free space)

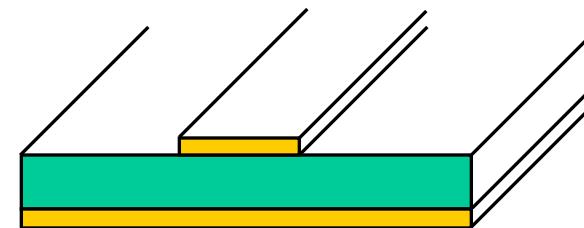
- **TEM (transverse electromagnetic) waveguides = transmission lines:**

Examples : copper wires, coaxial cables, microstrip lines

Such waveguides enable **the baseband as well as the carrier frequency transmissions.**
No low cutoff frequency...



Coaxial cable



Microstrip line

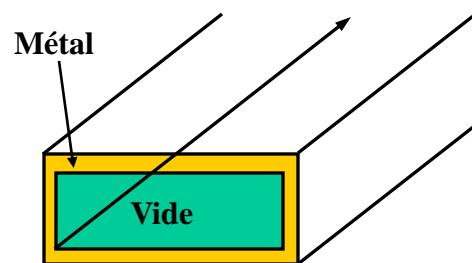
Introduction

The constraints which the transmission system is submitted to

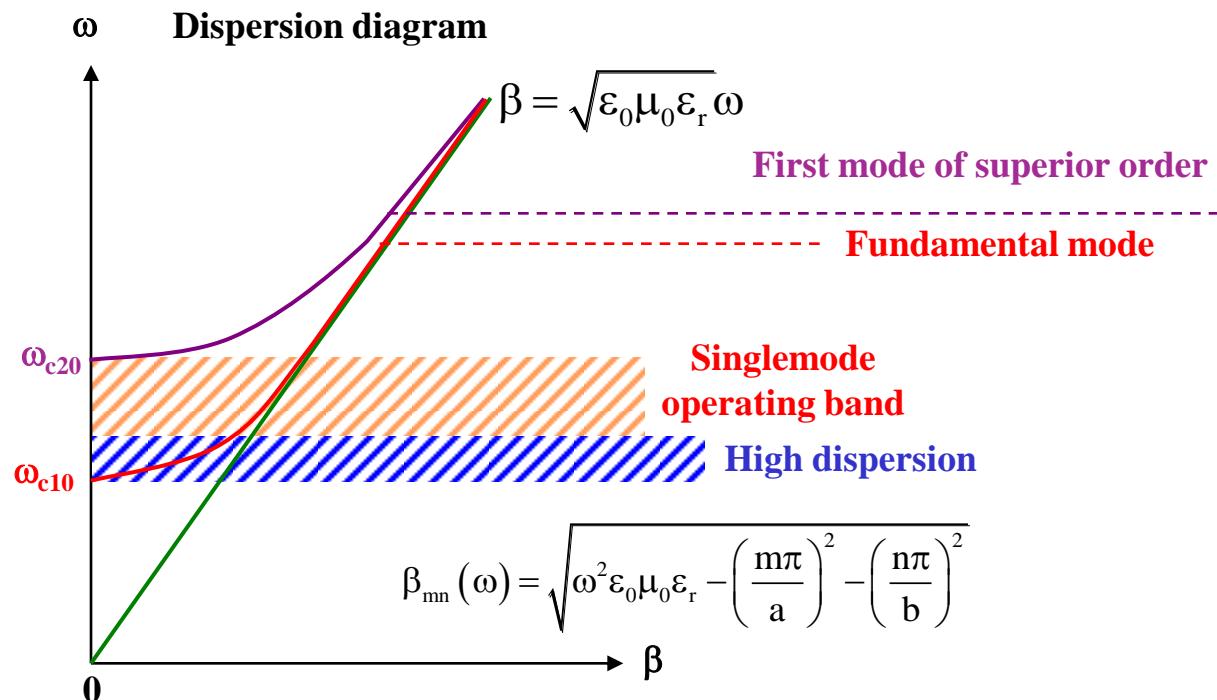
- Non TEM waveguides:

Examples: rectangular metallic waveguides, cylindrical metallic waveguides

Such waveguides only enable **carrier frequency transmissions** because of their low cutoff frequency. They also feature a maximum frequency of use (these frequencies depend on the dimensions of the considered waveguide).



Rectangular
metallic
waveguide

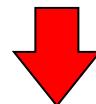


To each propagation mode corresponds a cutoff frequency ω_c (frequency under which the mode does not propagate : $\beta = 0$).

- **Free space:** the **transmission** has to be performed over a carrier frequency.

Example: how to transmitt sound in free space?

Acoustic waves (mechanical waves) undergo high attenuation,
have low propagation speed and require that the Tx and Rx be in direct visibility.



An acoustic wave can be converted into a low frequency electric signal
means of a microphone (voice $\cong 4\text{kHz}$).



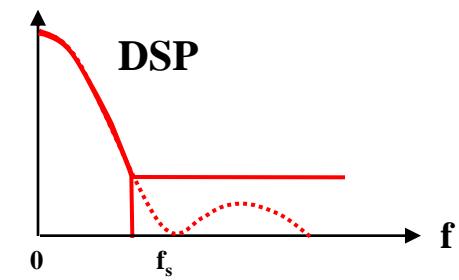
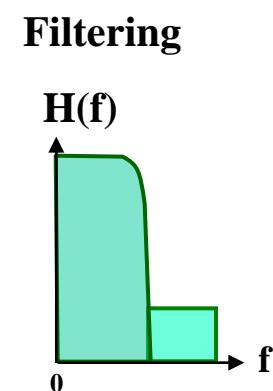
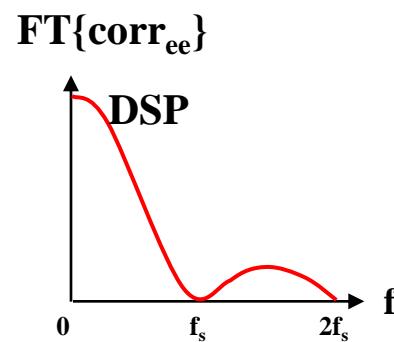
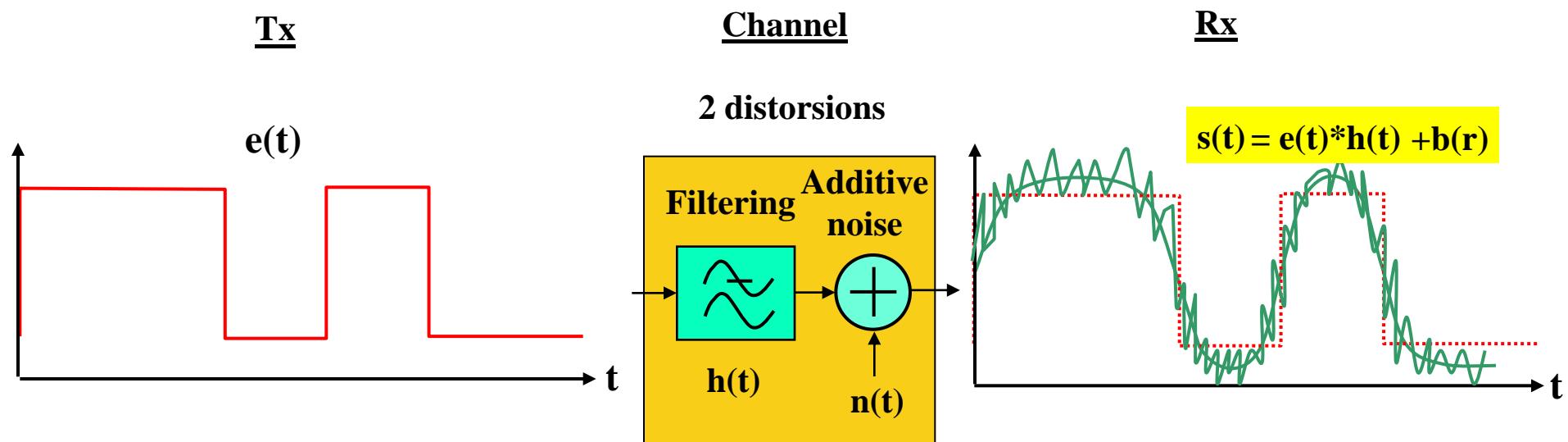
The free space transmission of a sound information converted into an EM wave
(digital or not) requires the use of a carrier frequency.

Indeed, the dimensions of the antennas are in the order of $\lambda/2$.
Consequently the transmission of low frequency EM signals would require
extremely large antennas ($1\text{ kHz} < f < 100\text{ kHz} \Leftrightarrow 300\text{ km} > \lambda = 3\text{km}$).

Introduction

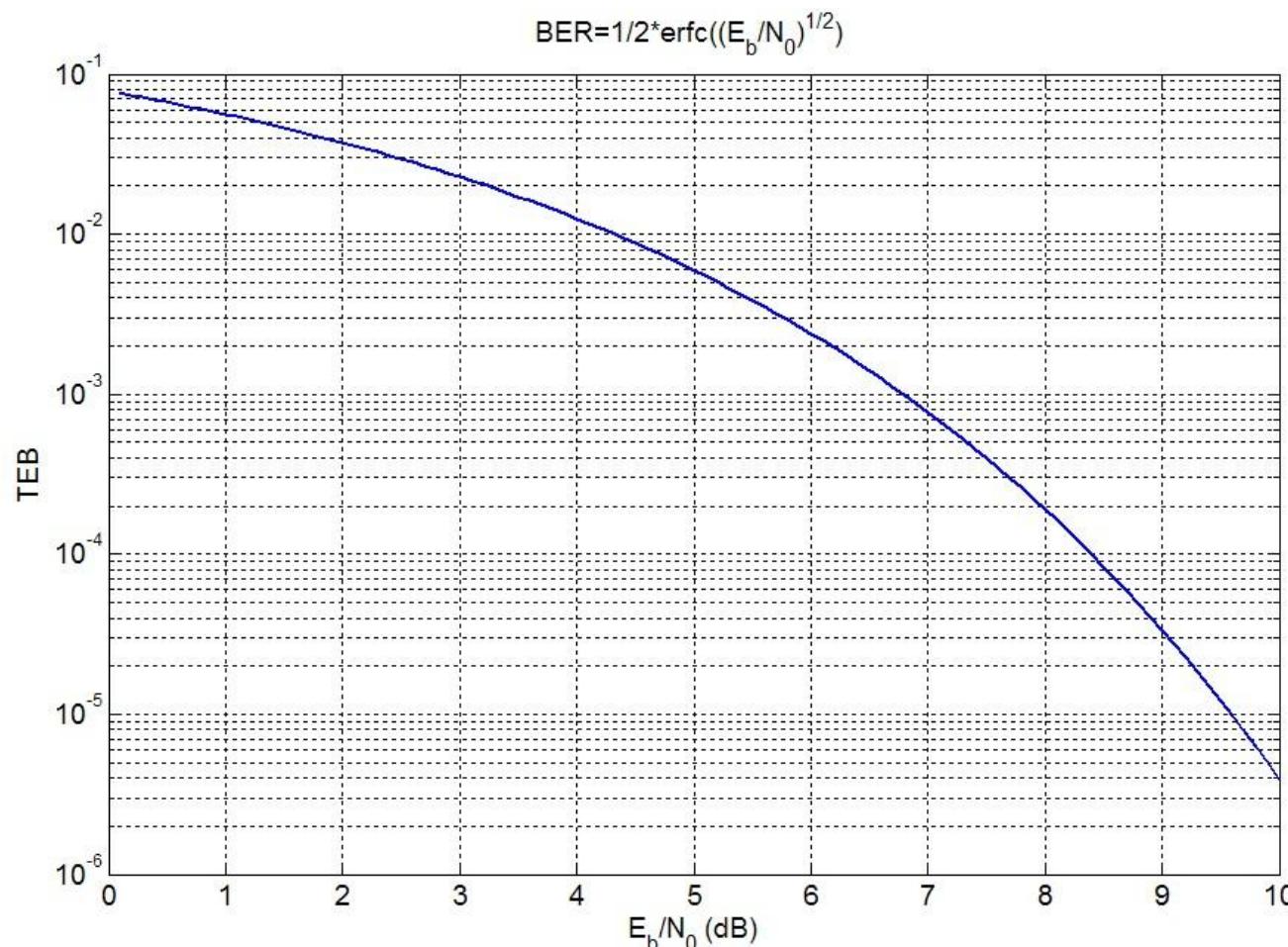
Origin of the errors

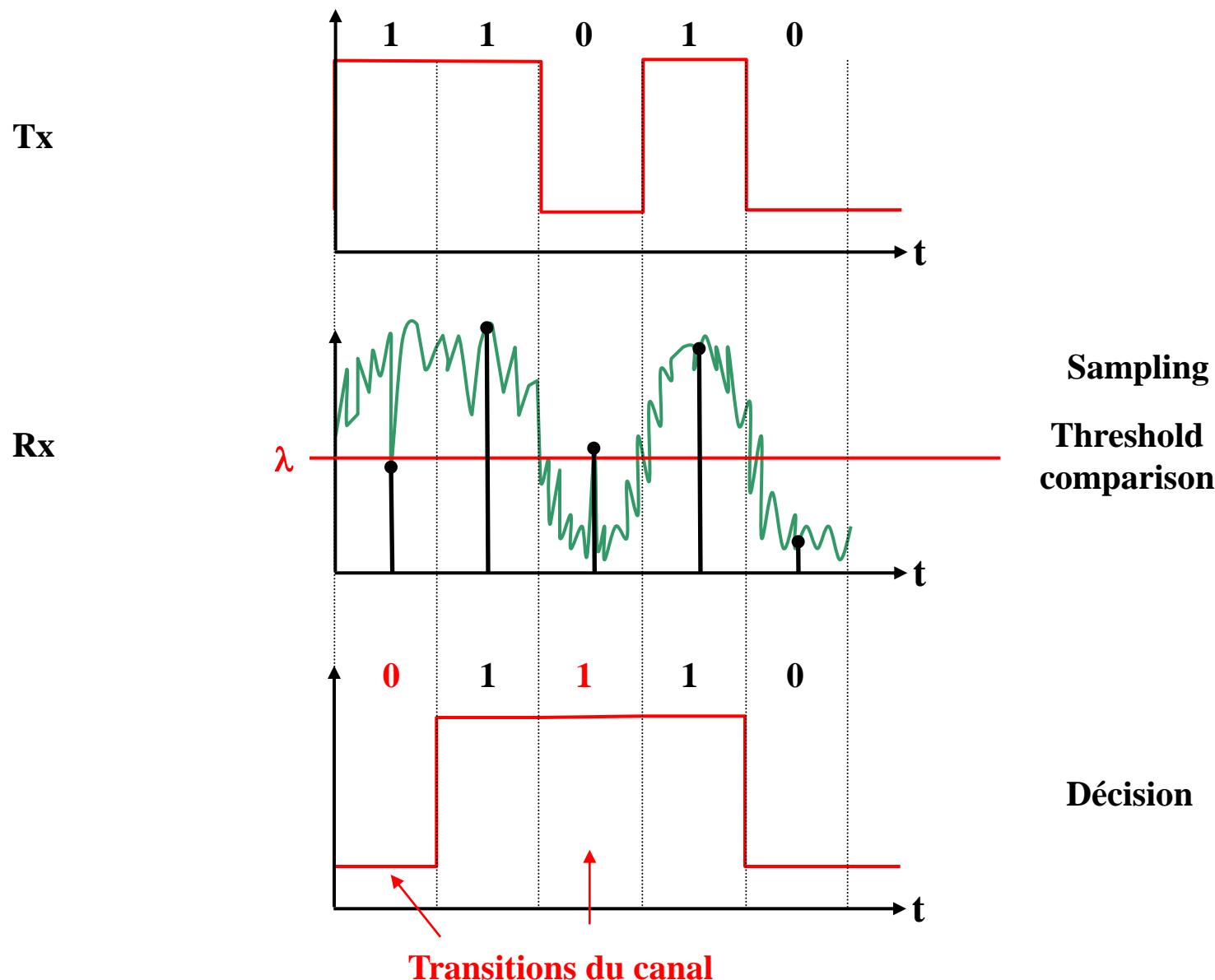
➤ Baseband transmission



Introduction

Origin of the errors



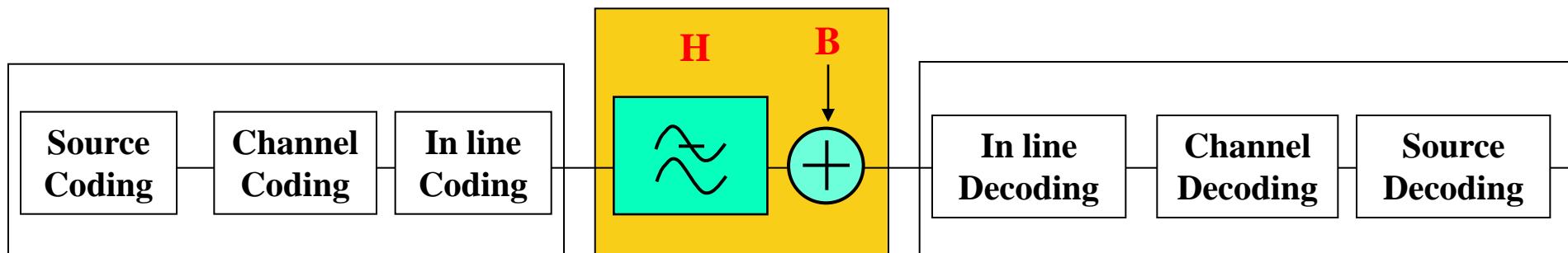


Introduction

Constraints and DOFs of digital communication systems

Constraints	DOFs	
	Baseband transmission	Tr. over a carrier freq
Bit rate	Source coding	
BER	Channel sounding Channel coding S/N In line coding (Nyquist criterion satisfaction)	Modulation format
Spectral width	In line coding	Modulation format
Spectral efficiency (bit/s/Hz)		Modulation format

Baseband Transmission



- ➔ **Source coding : information compression**, or how to transmitt as little digital data as possible (by coding symbols thanks to words of variable length, Huffman algorithm)
- ➔ **Channel coding : redundancy** for **correcting errors** provoked in reception by the additive noise of the channel (goal : improving the BER).
- ➔ **In line coding** : Choosing the shape of the pulses representing the digits. One of the choice criteria of the in line code is the ratio spectrum width/bit rate.

Coding without repetition

Symbols	Coded word	Received word	Transitions detection	Errors correction
0	0	0	NON	NON
1	1	1	NON	NON

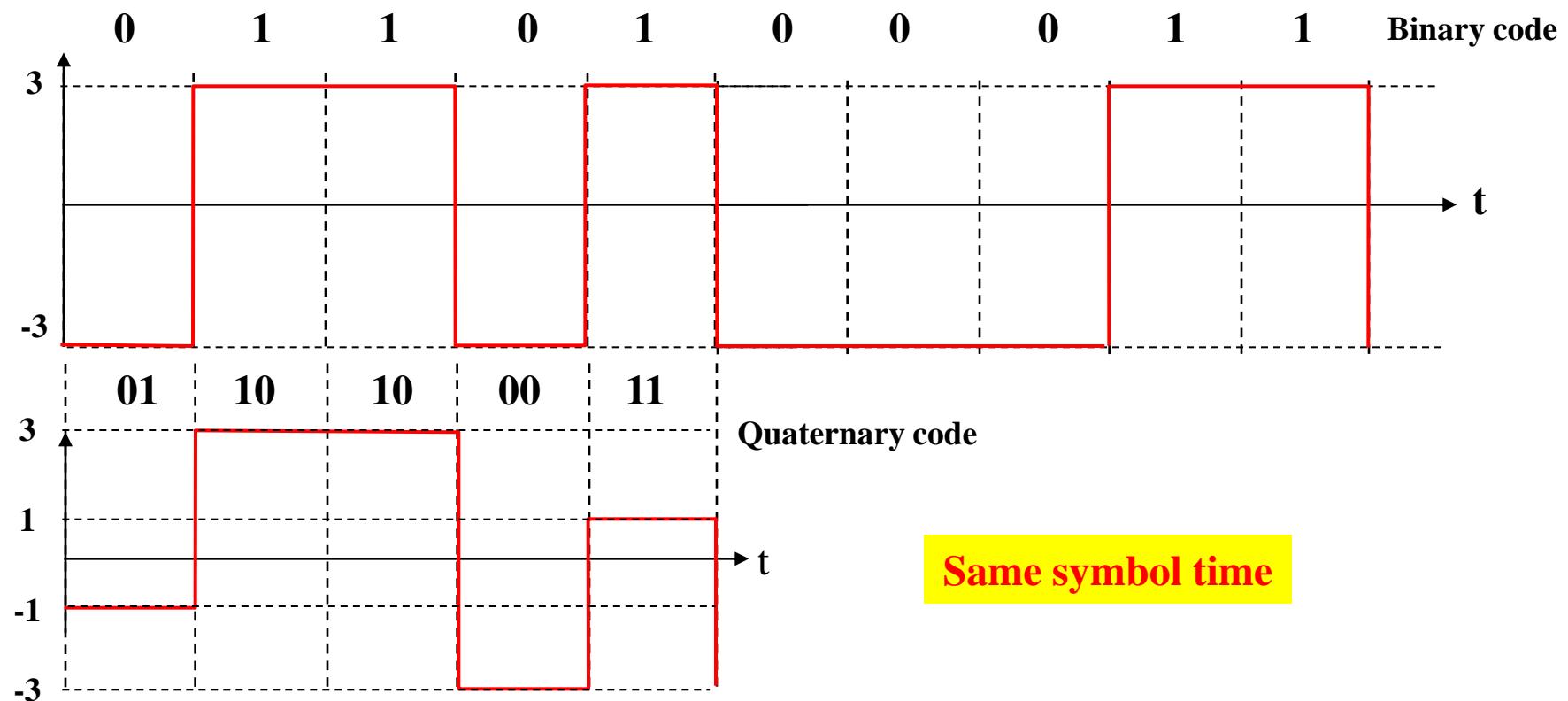
Coding with double repetition

Symbols	Coded word	Received word	Transitions detection	Errors correction
0	00	00	NON	NON
1	11	01	OUI	NON
		10	OUI	NON
		11	NON	NON

Coding with triple repetition

Symbols	Coded word	Received word	Transitions detection	Errors correction
0	000	000	NON	NON
1	111	001	OUI	OUI
		010	OUI	OUI
		011	OUI	OUI
		100	OUI	OUI
		101	OUI	OUI
		110	OUI	OUI
		111	NON	NON

Comparison of binary and quaternary coding :



Expression of the signal $e(t)$ out of the in line encoder :

Let us assume an **M-ary data (M-level) code** :

The encoder can transmit **$M = 2^n$ signals $S_i(t)$ that last T (symbol time).**

The transmitted signal is $e(t) = \sum_k S_{i(k)}(t - kT) ; i(k) = 0, 1, \dots, (M - 1)$

where $\begin{cases} k \text{ varies from } -\infty \text{ to } +\infty \\ i(k) \text{ is a fct}^\circ \text{ of the symbol transmitted at } kT \text{ time} \end{cases}$

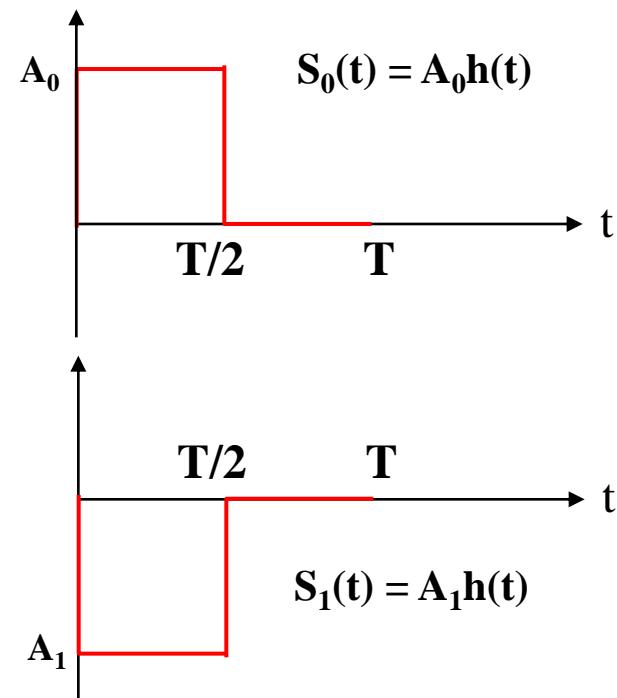
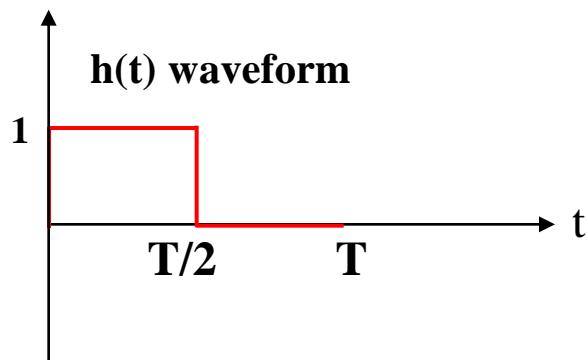
For most codes, the $S_{i(k)}(t)$ signals may be expressed as a **fct^o of a unique waveform $h(t)$** (see fig.) :

$$S_{i(k)} = a_{i(k)} h(t - kT) \quad \text{where } h(t) \text{ is the unique waveform}$$

Baseband transmission

In line coding

Exple : Binary signal based on a unique waveform



Thus

$$e(t) = \sum_k a_{i(k)} h(t - kT)$$

where the $a_{i(k)}$ values belong to an M -elements alphabet

$$\{A_0, A_1, \dots, A_{M-1}\}$$

Criteria for choosing an in line code :

- ➔ Modulation speed
- ➔ Noise sensitivity
- ➔ Spectral efficiency
- ➔ Clock recovery at the receiver (dependent on the spectrum of the code : it is interesting that the latter exhibits a pic at the clock frequency)



The spectral characteristics are important

Power spectral density of an in line code :

Rq. : In this course the PSD is also called the « spectrum ».

Expression of the PSD of an in line code :

$$e(t) = \sum_k a_{i(k)} h(t - kT) \quad \text{which we write} \quad e(t) = \sum_k a_k h(t - kT)$$

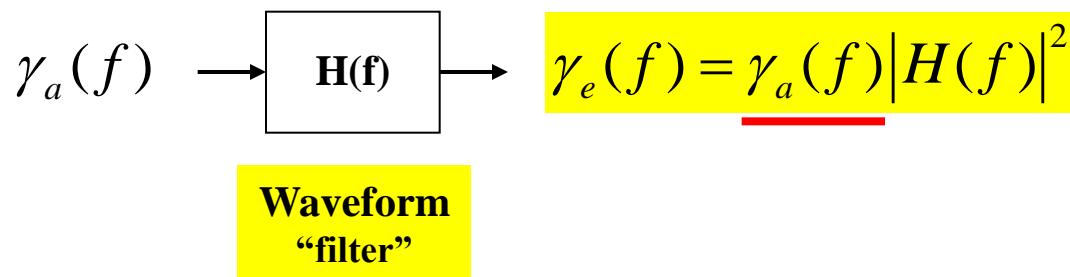
\uparrow
 $i(k) = 0, 1, \dots, (M - 1)$

Rq. : **$e(t)$ may be considered as the filtering of an $a(t)$ signal** $a(t) = \sum_k a_k \delta(t - kT)$
by a filter of impulse response $h(t)$

$$a(t) = \sum_k a_k \delta(t - kT) \rightarrow \boxed{h(t)} \rightarrow e(t) = a(t) * h(t)$$

Waveform
“filter”

Let : $\begin{cases} \gamma_e(f) & \text{be the PSD of } e(t) \\ \gamma_a(f) & \text{be the PSD of } a(t) = \sum_k a_k \delta(t - kT) \\ H(f) & \text{be the FT of } h(t) \\ & \text{(transfer function)} \end{cases}$



$\gamma_a(f)$ is a function of the frequency that is only dependent of the statistical properties of the a_k symbols.

What is the value of $\gamma_a(f)$? (Cf Glavieux p. 14-15 and annexe I)

$$\gamma_a(f) = \frac{\sigma_a^2}{T} + \frac{2\sigma_a^2}{T} \sum_{k=1}^{\infty} \Gamma'_a(k) \cos(2\pi f T) + \frac{m_a^2}{T^2} \sum_{k=-\infty}^{+\infty} \delta(f - \frac{k}{T})$$

avec $\begin{cases} m_a & \text{mean} \\ \sigma_a^2 & \text{variance} \\ \Gamma'_a(k) & \text{normalized autocorrelation fct° of the centered } a_k \text{ symbols.} \end{cases}$

The a_k symbols **being stationnary** (their statistical properties do not vary with time).

$$\left\{ \begin{array}{l} m_a = E[a_n] \forall n \quad \text{expectation = statistical average} \\ \sigma_a^2 = E[(a_n - m_a)^2] \forall n \\ \Gamma'_a(j) = \frac{E[(a_n - m_a)(a_{n-k} - m_a)]}{\sigma_a^2} \forall n, k \end{array} \right.$$

After multiplying $\gamma_a(f)$ by $|H(f)|^2$, the PSD of $\gamma_e(f)$ may be expressed as

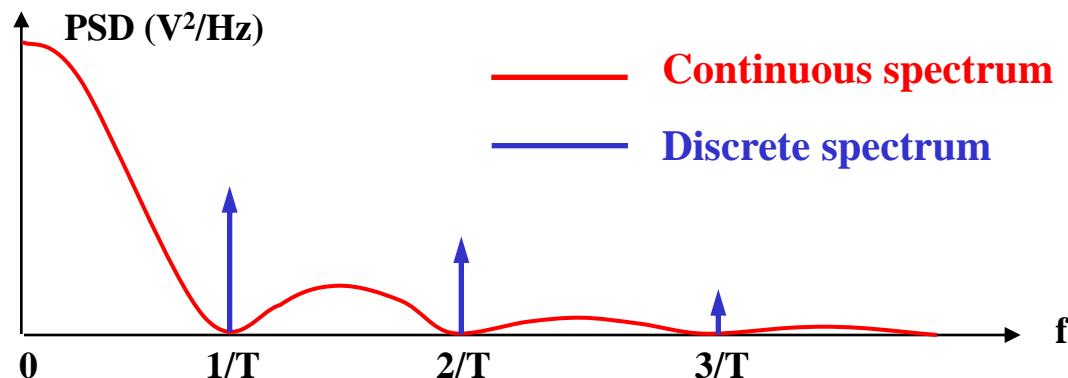
the **summ of a continuous spectrum** $\gamma_e^c(f)$

$$\gamma_e^c(f) = \frac{\sigma_a^2}{T} |H(f)|^2 + \frac{2\sigma_a^2}{T} |H(f)|^2 \sum_{k=1}^{\infty} \Gamma_a(k) \cos(2\pi fT)$$

and of a **discrete spectrum**

$$\gamma_e^d(f)$$

$$\gamma_e^d(f) = \frac{m_a^2}{T^2} \sum_{k=-\infty}^{+\infty} \left| H\left(\frac{k}{T}\right) \right|^2 \delta(f - \frac{k}{T})$$



$$\gamma_e^c(f) = \frac{\sigma_a^2}{T} |H(f)|^2 + \frac{2\sigma_a^2}{T} |H(f)|^2 \sum_{k=1}^{\infty} \Gamma_a(k) \cos(2\pi f T) \quad \gamma_e^d(f) = \frac{m_a^2}{T^2} \sum_{k=-\infty}^{+\infty} \left| H\left(\frac{k}{T}\right) \right|^2 \delta(f - \frac{k}{T})$$

Rq. : The expression of the spectrum of an in line code is very simplified :

- If the a_k symbols are independent

$$\Gamma_a(k) = cc_{a_k} = 0 \rightarrow \gamma_e^c(f) = \frac{\sigma_a^2}{T} |H(f)|^2$$

- If the mean m_a of the a_k symbols is zero

$$\rightarrow \gamma_e^d(f) = 0$$

If those two conditions are respected then :

$$\gamma_e(f) = \frac{\sigma_a^2}{T} |H(f)|^2$$

Where $\sigma_a^2 = E[(a_n - m_a)^2] = E[a_n^2]$ because m_a is zero

To remember : The PSD of an in line code is dependant on

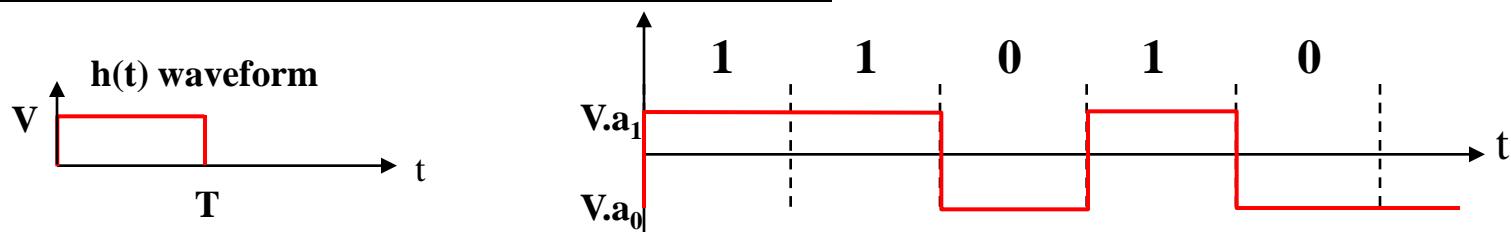
- ➔ the FT of the $h(t)$ waveform
- ➔ the statistical properties of the a_k symbols

Exemples of in line codes :

- In line codes with independent symbols (not correlated)
- In line codes with dependent symbols (correlated)

- independent symbols (uncorrelated):

The binary NRZ (Non Return to Zero code) :



Statistical characteristics of the data

$$b_1 \rightarrow a_1 = 1; b_0 \rightarrow a_0 = -1 \quad \text{where } b_i \text{ is the considered bit}$$

$$m_a = p_1 a_1 + p_0 a_0 = \frac{1}{2} 1 + \frac{1}{2} (-1) = 0$$

!!! The calculation is done without taking the $*h(t)$ waveform into account

$$m_a = 0$$

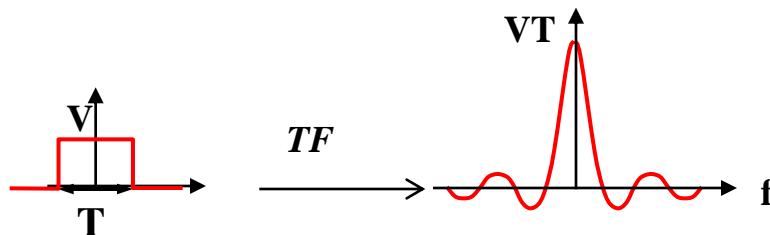
$$\sigma_a^2 = E[(a_n - m_a)^2] = E[a_n^2] = \sum_{n=1}^N P_n a_n^2 = \frac{1}{2} 1 + \frac{1}{2} 1 = 1 \quad \sigma_a^2 = 1$$

- Independent symbols
- The mean m_a of the a_k symbols is zero

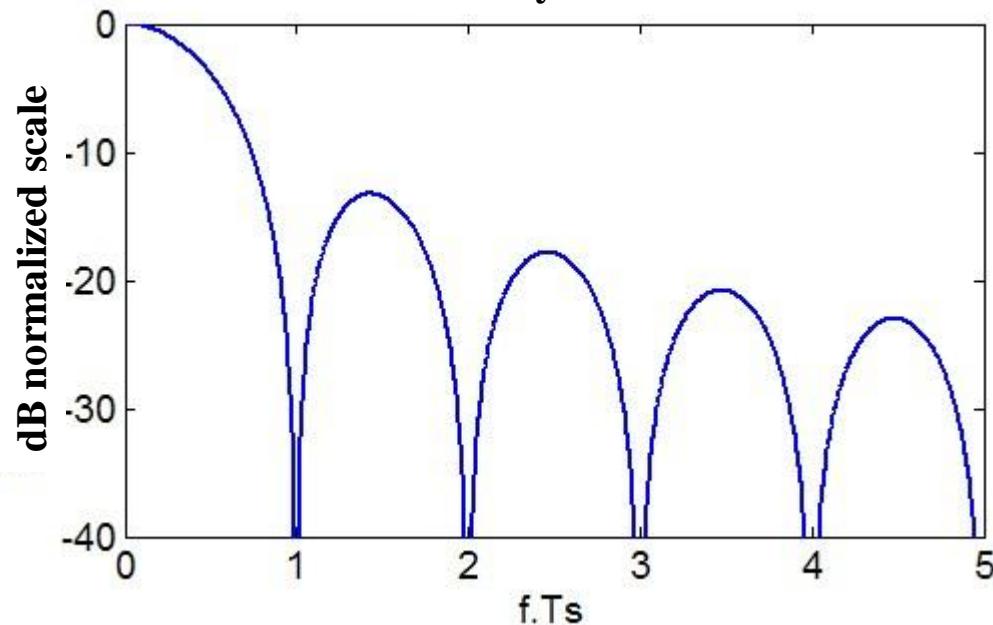
$$\Rightarrow \gamma_e(f) = \frac{\sigma_a^2}{T} |H(f)|^2$$

$$h(t) = V \cdot \Pi_T(t) \xrightarrow{TF} H(f) = VT \frac{\sin(\pi fT)}{\pi fT}$$

$$\Rightarrow \gamma_e(f) = V^2 T \frac{\sin^2(\pi fT)}{(\pi fT)^2}$$



Binary NRZ

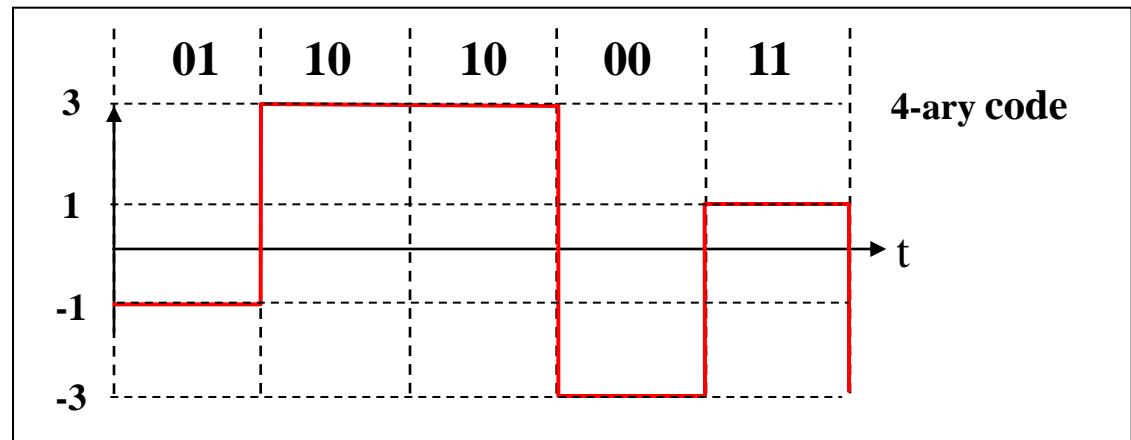
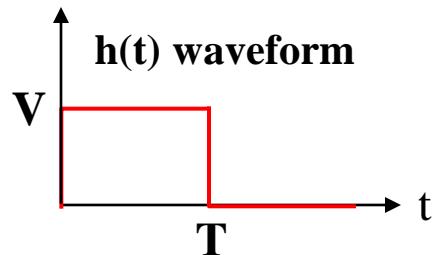


The PSD of a binary NRZ code vanishes for every multiple of $1/T$ where T is the symbol time

M-ary NRZ code :

Let us consider an M-ary NRZ code whose a_k values belong to an M elements alphabet

$$a_k \in \{\pm 1, \pm 3, \dots, \pm(2p+1), \dots, \pm(M-1)\}$$



$$S_k = a_k \cdot V \cdot h(t)$$

M-ary NRZ code :

Let us consider an M-ary NRZ code whose a_k values belong to an M elements alphabet

$$a_k \in \{\pm 1, \pm 3, \dots, \pm(2p+1), \dots, \pm(M-1)\}$$

(example 4-ary code)

$$symb_{00} \rightarrow a_{00} = -3; symb_{01} \rightarrow a_{01} = -1; symb_{11} \rightarrow a_{11} = 1 symb_{10} \rightarrow a_{10} = 3$$

Statistical characteristics of the data

$$m_a = E[a_k] = \sum_k a_k p_k = \sum_{p=-(M-2)/2}^{(M-2)/2} (2p+1) \frac{1}{M} = 0$$

!!! The calculation is done without taking the $*h(t)$ waveform into account

$m_a = 0$

$$\sigma_a^2 = E[a_n^2] = \frac{2}{M} \sum_{p=0}^{(M/2)-1} (2p+1)^2 \Rightarrow \sigma_a^2 = \frac{M^2 - 1}{3}$$

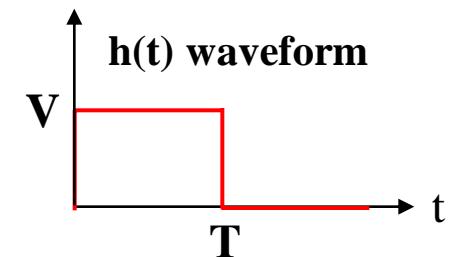
M-ary NRZ code :

- Independent symbols
- The mean m_a of the a_k symbols is zero

$$\Rightarrow \gamma_e(f) = \frac{\sigma_a^2}{T} |H(f)|^2$$

where $\sigma_a^2 = \frac{M^2 - 1}{3}$

The waveform $h(t)$ is still a rectangle of amplitude V and of symbol time T .



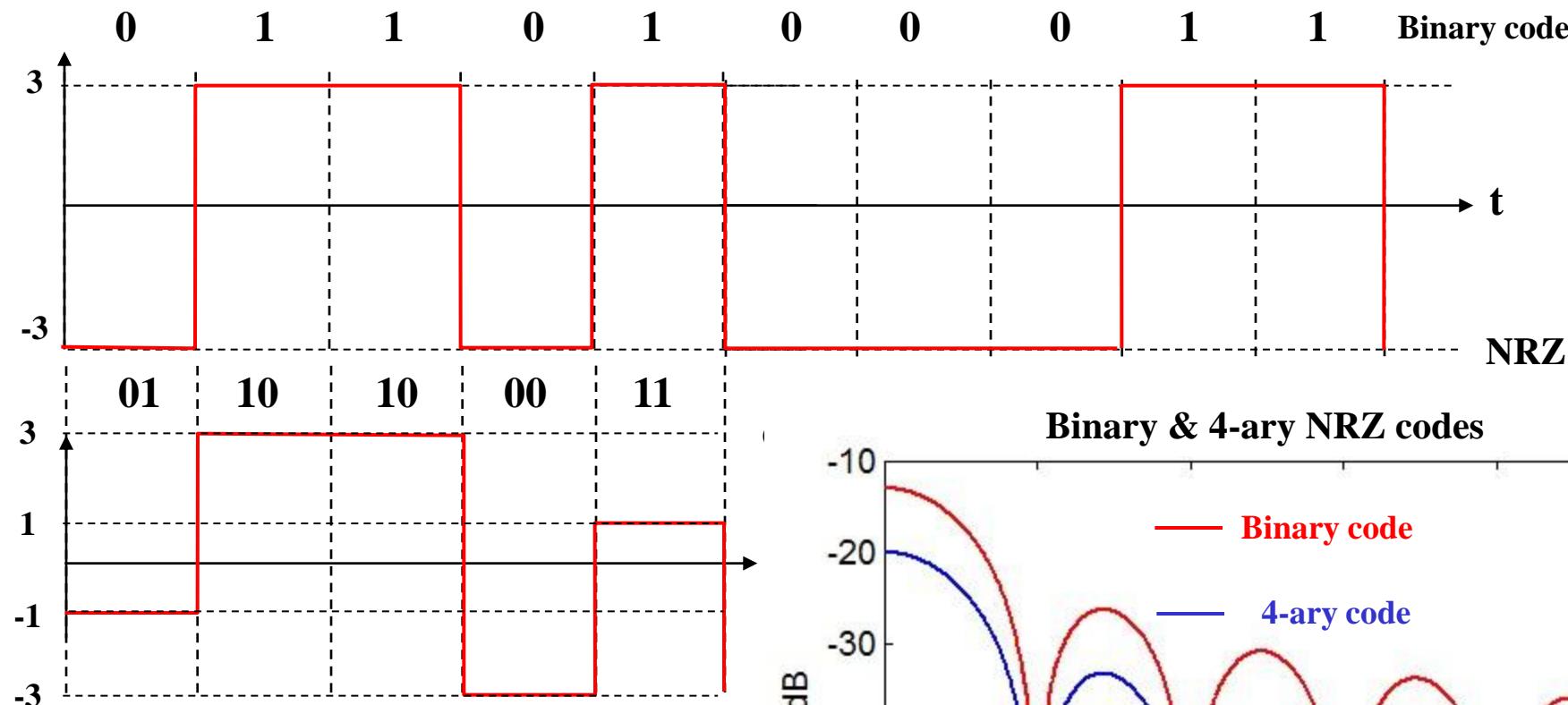
$$h(t) \xrightarrow{Tf} H(f) = VT \frac{\sin(\pi fT)}{\pi fT}$$

$$\Rightarrow \gamma_e(f) = \frac{M^2 - 1}{3} V^2 T \frac{\sin^2(\pi fT)}{(\pi fT)^2}$$

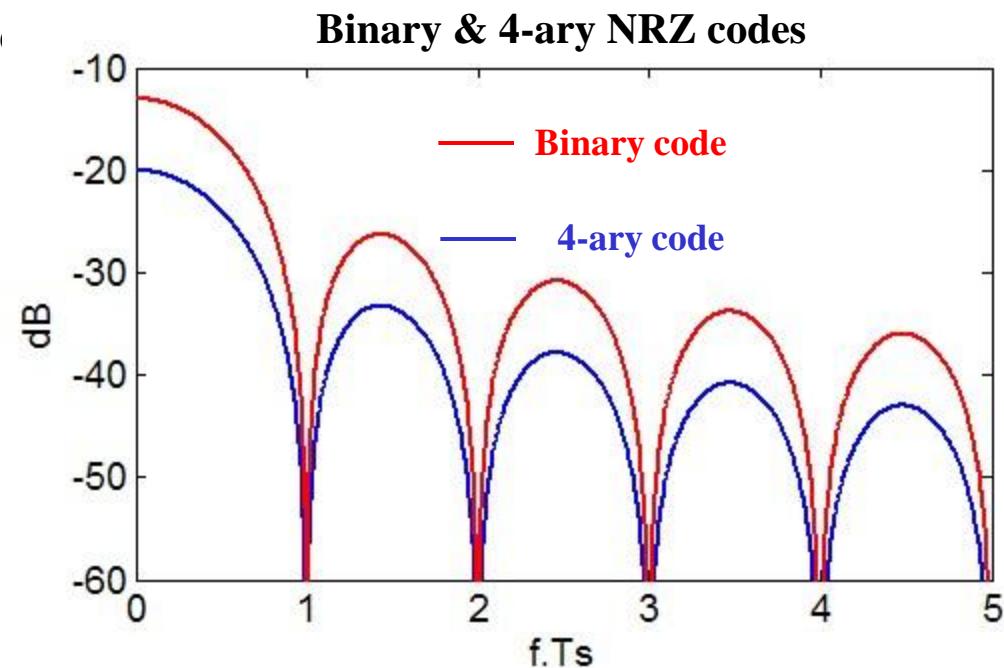
Baseband transmission

In line coding

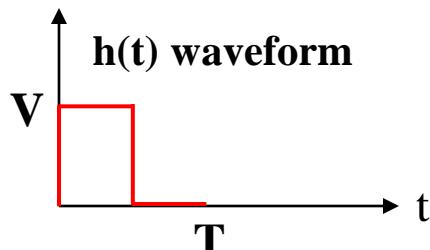
Comparison of binary and 4-ary NRZ codes :



For a same symbol time, a 4-ary code enables transmitting in the same bandwidth a $n = \log_2(M)$ times higher bit number than a binary code. Nevertheless this code is more noise sensitive.

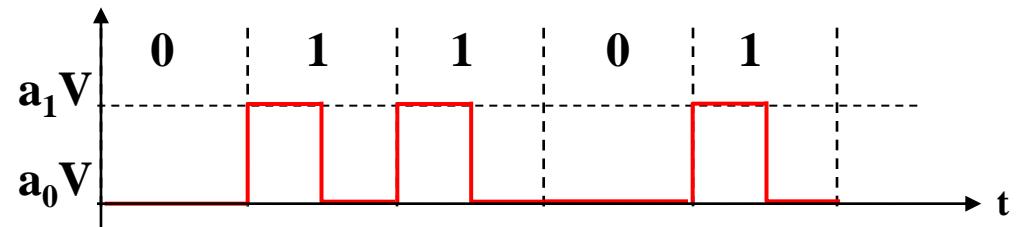


The RZ binary code :



$$a_k \in \{0,1\}$$

$$S_k = a_k \cdot V \cdot h(t)$$



The waveform $h(t)$ is a T long signal consisting in λT ($0 < \lambda < 1$) long rectangle followed by a $(1-\lambda)T$ long return to zero.

Statistical characteristics of the data

$$m_a = p_1 a_1 + p_0 a_0 = \frac{1}{2} 1 + \frac{1}{2} 0 = \frac{1}{2}$$

$$m_a = \frac{1}{2}$$

$$\sigma_a^2 = E[(a_k - m_a)^2] = E[a_k^2 - 2m_a a_k + m_a^2] = E[a_k^2] - 2m_a E[a_k] + E[m_a^2]$$

$$\sigma_a^2 = E[a_k^2] - 2m_a^2 + m_a^2 \Rightarrow \sigma_a^2 = E[a_k^2] - m_a^2$$

$$\Rightarrow \sigma_a^2 = 1^2 p_1 - 0^2 p_0 - \left(\frac{1}{2}\right)^2 = \frac{1}{2} - \frac{1}{4}$$

$$\sigma_a^2 = \frac{1}{4}$$

We have

$$m_a = \frac{1}{2} \quad \sigma_a^2 = \frac{1}{4} \quad H(f) = V\lambda T \frac{\sin(\pi f \lambda T)}{\pi f \lambda T} \quad \lambda = \frac{1}{2}$$

Continuous part of the spectrum

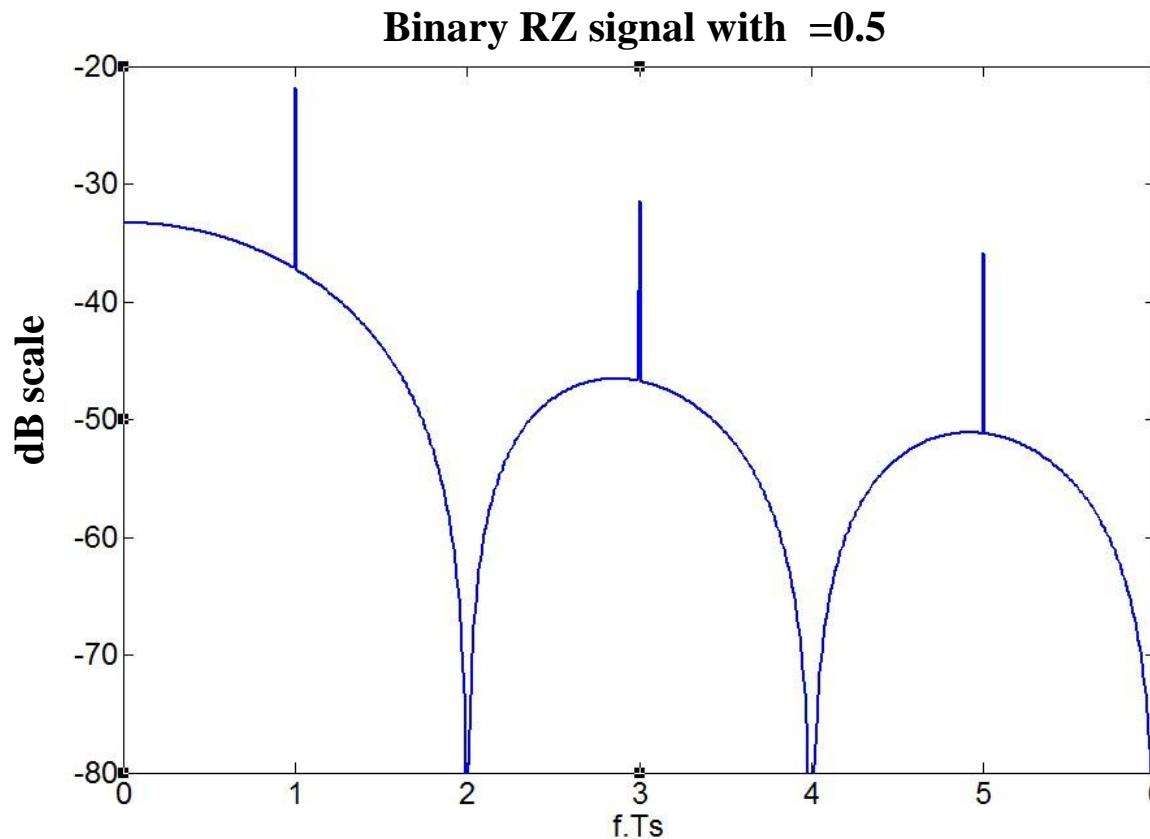
$$\gamma_e^c(f) = \frac{\sigma_a^2}{T} |H(f)|^2 \Rightarrow \gamma_e^c(f) = \frac{\sigma_a^2}{T} V^2 \lambda^2 T \frac{\sin^2(\pi f \lambda T)}{(\pi f \lambda T)^2} \Rightarrow \gamma_e^c(f) = \frac{1}{16} V^2 T \frac{\sin^2(\pi f T / 2)}{(\pi f T / 2)^2}$$

Discrete part of the spectrum

$$\gamma_e^d(f) = \frac{m_a^2}{T^2} \sum_{k=-\infty}^{+\infty} \left| H\left(\frac{k}{T}\right) \right|^2 \delta(f - \frac{k}{T}) \Rightarrow \gamma_e^d(f) = \frac{m_a^2}{T^2} \lambda^2 V^2 T^2 \sum_{k=-\infty}^{+\infty} \left(\frac{\sin(k \lambda \pi)}{k \lambda \pi} \right)^2 \delta(f - \frac{k}{T})$$

$$\Rightarrow \gamma_e^d(f) = \frac{1}{16} V^2 \sum_{p=-\infty}^{+\infty} \frac{1}{(2p+1)^2 \pi^2} \delta(f - \frac{2p+1}{T})$$

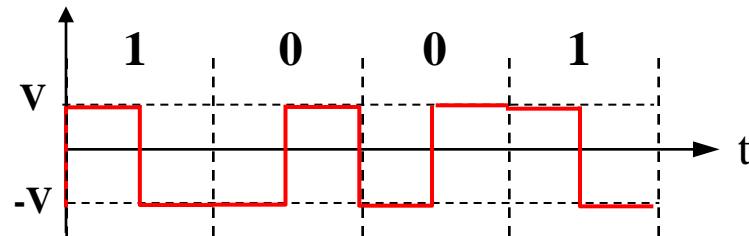
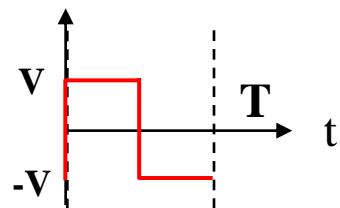
The spectrum of an RZ signal is the superposition of a continuous spectrum and of a discrete spectrum (odd harmonics of $1/T$)



The pic at $/T_s$ is useful for the clock recovery at the receiver.

The biphase binary code (Manchester) :

Waveform $h(t)$



This in line code always presents $+V \square -V$ transitions (during each symbol time) that may facilitate the clock recovery.

$$m_a = p_1 a_1 + p_0 a_0 = \frac{1}{2}1 + \frac{1}{2}(-1) = 0 \quad \boxed{m_a = 0}$$

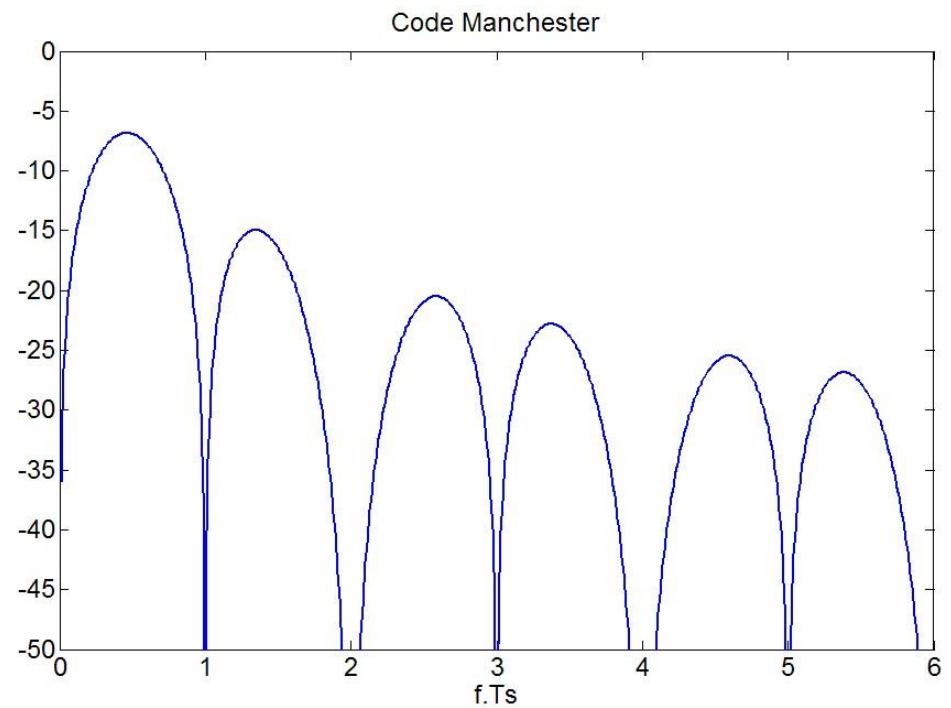
Rq : $m_a=0 \Rightarrow$ the spectrum has no discrete part

The biphase binary code (Manchester) :

$$\gamma_e(f) = V^2 T \sin^2\left(\frac{\pi f T}{2}\right) \sin_c^2\left(\frac{f T}{2}\right)$$

According to “A. Glavieux”...

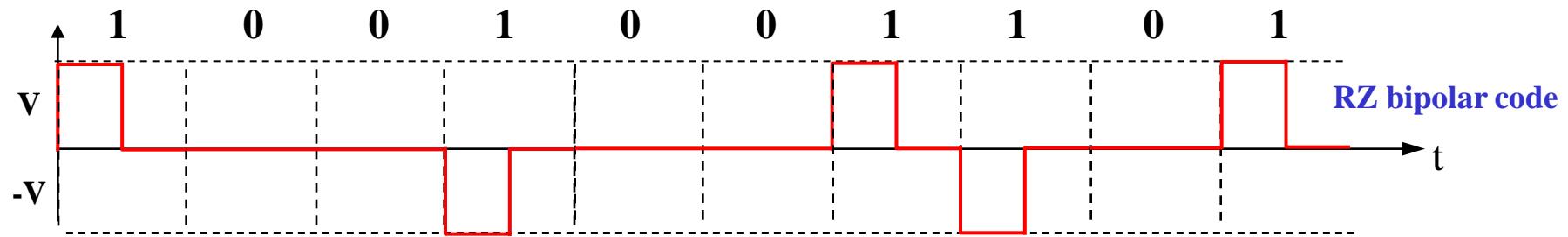
The PSD is 0 for $f = 0$



- In ligne codes with dependent symbols (correlated):

Bipolar code :

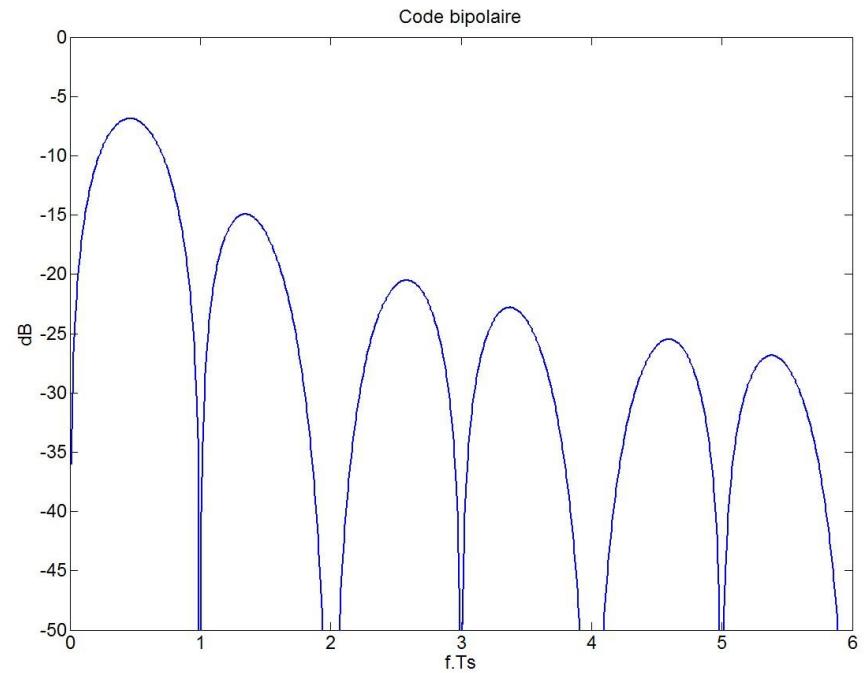
Two successive “1” are not independent (alternance rule)



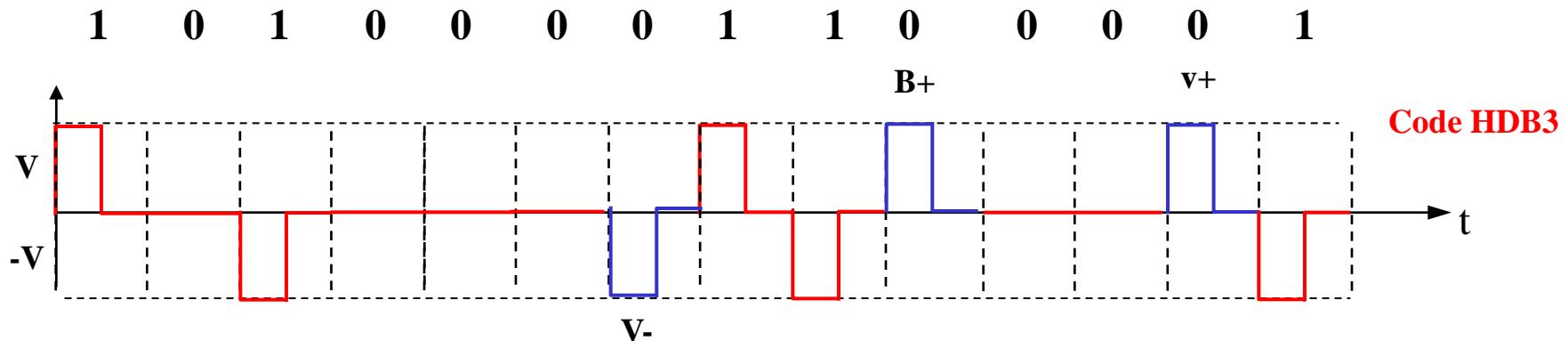
$$\gamma_e(f) = \frac{V^2 T}{4} \sin^2(\pi f T) \sin_c^2\left(\frac{f T}{2}\right)$$

According to “A. Glavieux”...

The PSD is 0 for $f=0$



Le code HDBn:



Bipolar code : alternance bipolar rule on the “1” transmitted

Bits de viol en $(n+1)^{\text{ième}}$ position (d'où le HDBn) lorsque $n+1$ “0” consécutifs sont transmis.
Les bits de viol respectent la règle d'alternance bipolaire.

Bits de bourrage afin de ne pas confondre un “1” avec un bit de viol

Intérêt : permettre la récupération d'horloge

Le HDB3 est utilisé dans le RTC (réseau téléphonique commuté)

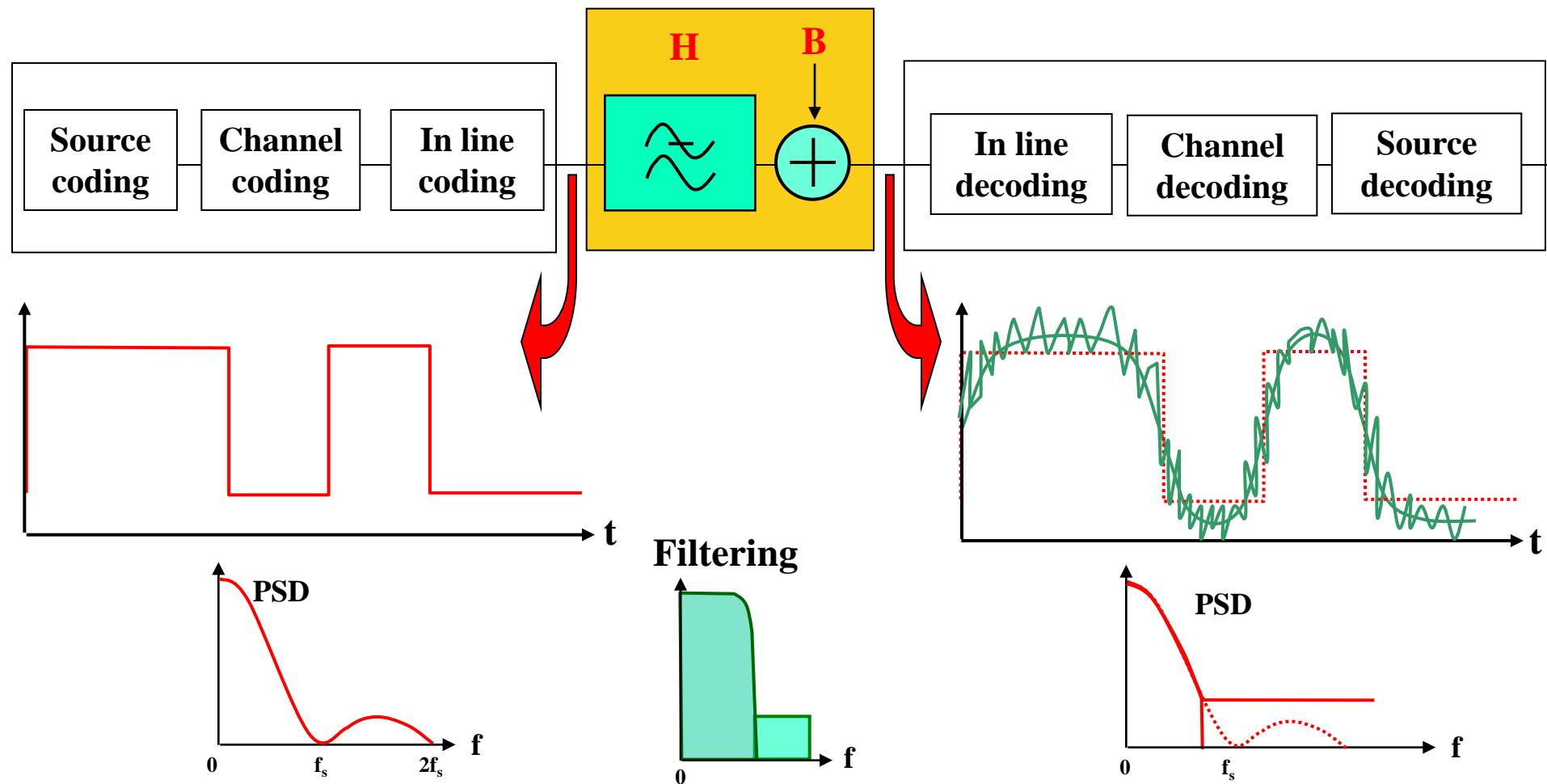
La DSP d'un HDB3 (RZ) est semblable à celle d'un code bipolaire RZ.

Baseband transmission

Channel perturbations

☰ 2 Channel perturbations

The filtering \Leftrightarrow Limited bandwidth channel
Additive noise



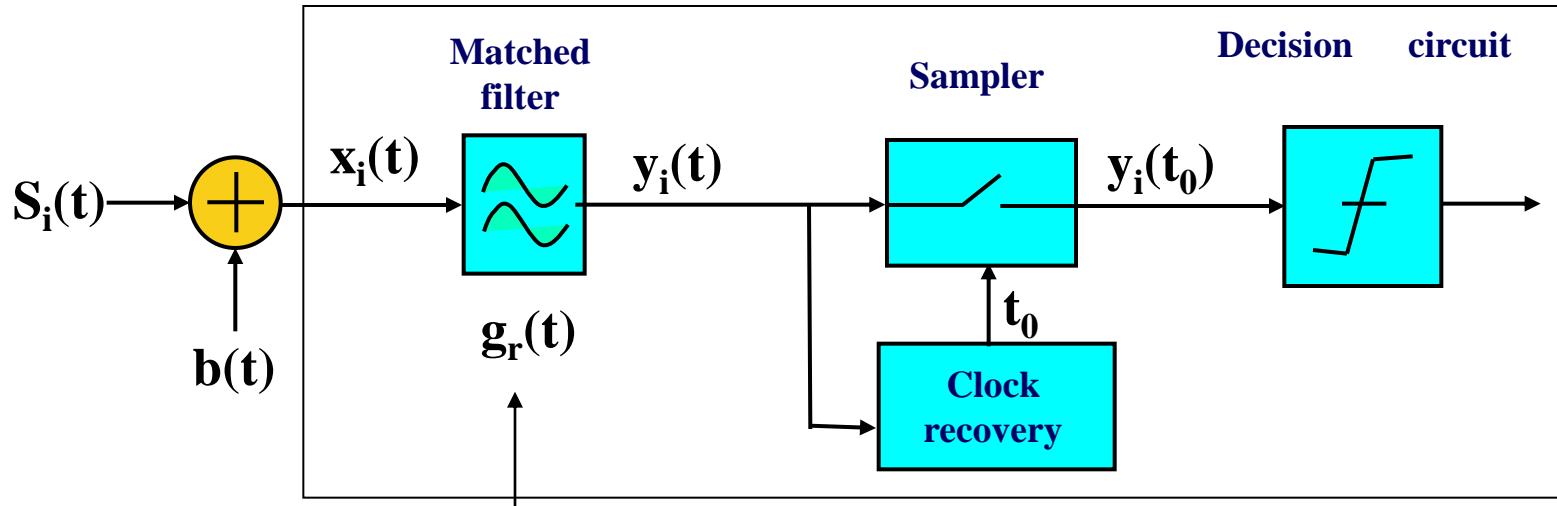
The filtering and the noise of the channel alleviate the BER at the receiver.

Let us consider a **channel with an unlimited bandwidth** (no filtering)

- BER at the receiver in an unlimited bandwidth channel**

One will try to quantify the amount of errors due to the noise of the channel

Receiver



Filters a maximum of the noise (white noise : its SPD is constant)

→ Filter input

$$x_i(t) = S_i(t) + b(t) \quad \text{where } b(t) \text{ is a white gaussian noise} \Rightarrow \gamma_b(f) = \frac{N_0}{2} \quad (\text{SDE})$$

i = 0,1

→ Filter output

$$y_i(t) = S_i(t) * g_r(t) + b(t) * g_r(t)$$

$n(t)$

avec $\sigma^2 = \frac{N_0}{2} \int_{-\infty}^{+\infty} |G_r(f)|^2 df$

```

graph LR
    N0[N0/2] --> Gr[Gr(f)]
    Gr --> SigmaSigma[σ²]
  
```

→ Sampler output

rms (root mean square) of the filtered noise

$$y_i(t_0) = S_i(t) * g_r(t) \Big|_{t_0} + n(t_0)$$

$$y_i(t_0) = U_i + n(t_0)$$

y_i(t₀) may be considered as a random gaussian function centered onto U_i with a σ standard deviation

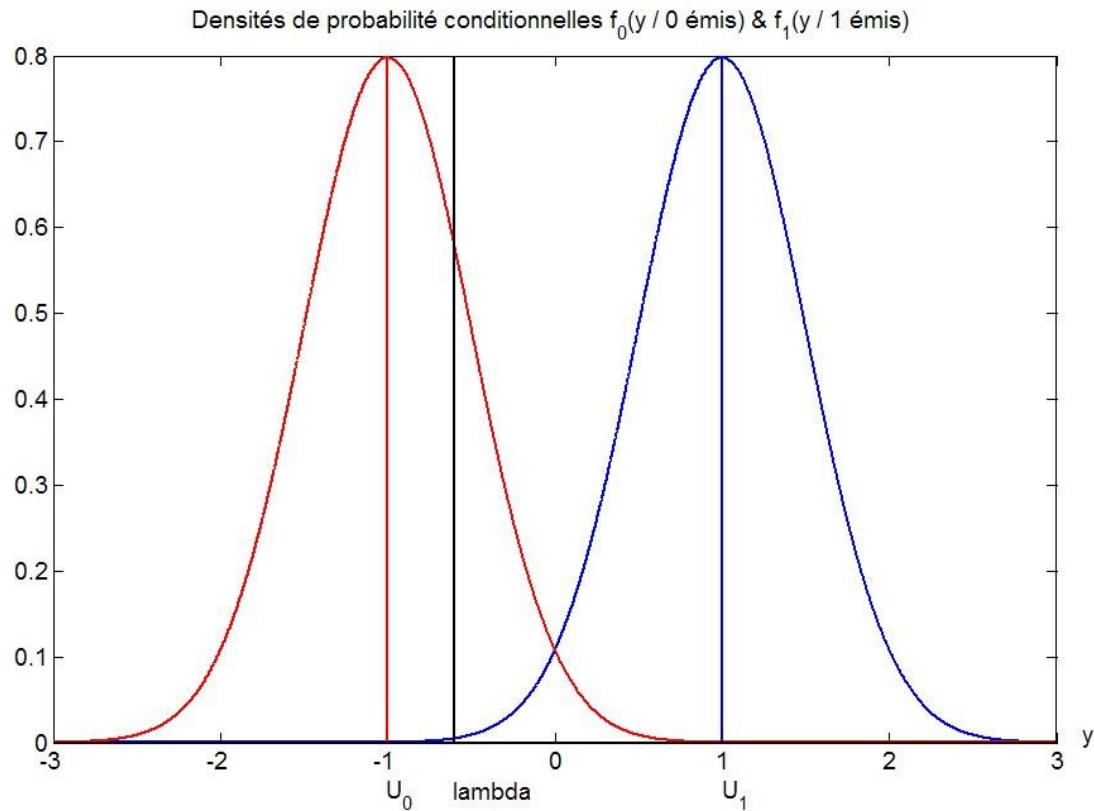


Deterministic value (i being supposed fixed)

Probability density functions of y_1 and y_0 at the sampler output

$$f_1(y) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left[-\frac{(y-U_1)^2}{\sigma^2 2}\right]$$

$$f_0(y) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left[\frac{(y-U_0)^2}{\sigma^2 2}\right]$$



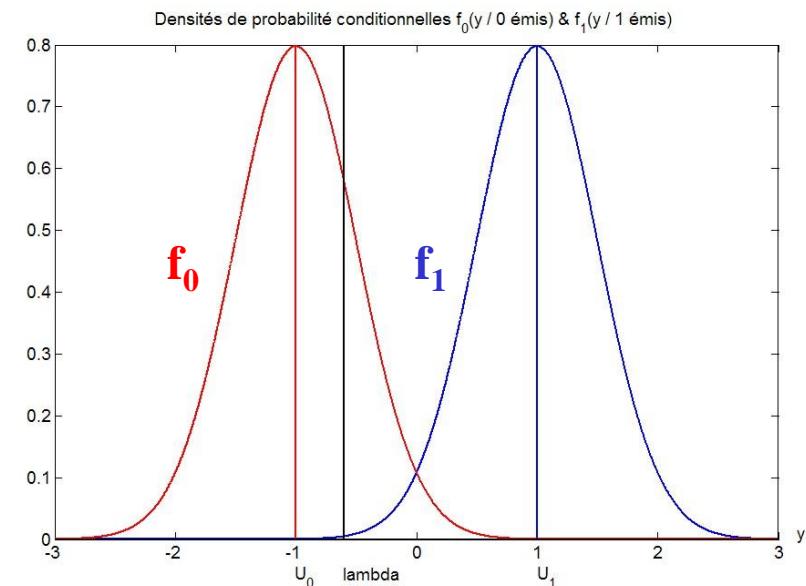
Error probability at the output of the decision device

Binary error probability

$$P_{be} = p_0 \cdot P(1/0 \text{ transmitted}) + p_1 \cdot P(0/1 \text{ transmitted})$$

↑
 Proba to decide “1” when a “0” is transmitted
 Proba to transmitt a “0”

$$P_{be} = p_0 \cdot \int_{-\infty}^{\lambda} f_0(y) dy + p_1 \cdot \int_{-\infty}^{\lambda} f_1(y) dy$$



After calculations and variable change :

$$P(0/1) = \frac{1}{2} \text{erfc} \left[-\frac{(\lambda - U_1)}{\sigma \sqrt{2}} \right]$$

$$P(1/0) = \frac{1}{2} \text{erfc} \left[\frac{(\lambda - U_0)}{\sigma \sqrt{2}} \right]$$

with

$$\text{erfc}(x) = \frac{2}{\sqrt{\pi}} \int_x^{\infty} \exp(-z^2) dz$$

$$\Rightarrow P_{eb} = \frac{1}{2} p_1 \text{erfc} \left[-\frac{(\lambda - U_1)}{\sigma \sqrt{2}} \right] + \frac{1}{2} p_0 \text{erfc} \left[\frac{(\lambda - U_0)}{\sigma \sqrt{2}} \right]$$

Optimal decision level

In case of **equiprobability** $p_0=p_1=1/2$, then

$$\lambda = \frac{U_0 + U_1}{2}$$

then

$$P_{eb} = \frac{1}{2} \text{erfc} \left[\frac{(U_1 - U_0)}{2\sigma \sqrt{2}} \right]$$

← Equiprobability
← Optimal decision level

- Optimal reception filter (matched filter)

The goal is to optimize the signal to noise ratio (erfc(x) is a decreasing function...) $\frac{U_1 - U_0}{2\sigma\sqrt{2}}$

by optimizing the noise filtering. **The transfer function of the optimal filter is the FT of pulses waveform.**

As a consequence

Pulse response of the matched filter :
$$g_r^{opt}(t) = k[S_1(t_0 - t) - S_0(t_0 - t)]$$

→ Let us express $\frac{U_1 - U_0}{2\sigma\sqrt{2}}$

$$\rightarrow U_1 - U_0 = [S_1(t) - S_0(t)] * k[S_1(t_0 - t) - S_0(t_0 - t)] \Big|_{t=t_0}$$

$$U_1 - U_0 = \int_{-\infty}^{+\infty} [S_1(\tau) - S_0(\tau)] \cdot k[S_1(\tau - t_0 + t) - S_0(\tau - t_0 + t)] d\tau \Big|_{t=t_0}$$

$$U_1 - U_0 = \int_{-\infty}^{+\infty} |S_1(\tau) - S_0(\tau)|^2 d\tau \quad \text{On assumes a real signal } (\)^2 = |\ |^2$$

According to Parseval : $\int_{-\infty}^{+\infty} |S_1(\tau) - S_0(\tau)|^2 d\tau = \int_{-\infty}^{+\infty} |\hat{S}_1(f) - \hat{S}_0(f)|^2 df = \underline{E_d} = U_1 - U_0$

ε of the \neq between the symbols

→ Let us express $2\sigma\sqrt{2} \dots \frac{U_1 - U_0}{2\sigma\sqrt{2}}$

$$\sigma^2 = \frac{N_0}{2} \int_{-\infty}^{+\infty} |G_r(f)|^2 df \xrightarrow{\text{Parseval}} \sigma^2 = \frac{N_0}{2} \int_{-\infty}^{+\infty} |g_r(t)|^2 dt$$

$$\Rightarrow \sigma^2 = \frac{N_0}{2} \int_{-\infty}^{+\infty} |S_1(t_0 - t) - S_0(t_0 - t)|^2 dt \quad \Rightarrow \sigma^2 = \frac{N_0}{2} E_d$$

Thus $\frac{U_1 - U_0}{2\sigma\sqrt{2}} = \sqrt{\frac{E_d}{4N_0}}$

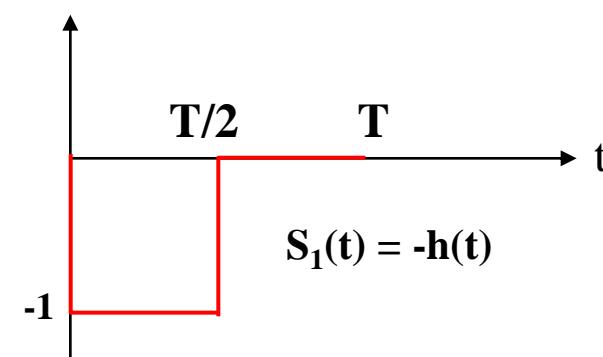
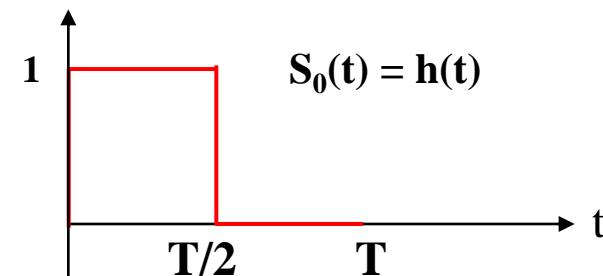
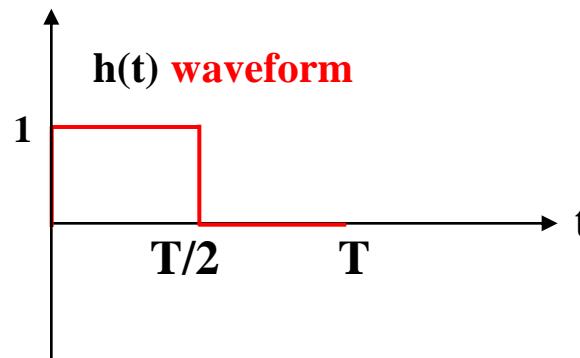
$$\Rightarrow P_{eb} = \frac{1}{2} \operatorname{erfc} \left(\sqrt{\frac{E_d}{4N_0}} \right)$$

← Équiprobability
← Optimal decision level

Baseband transmission

Channel perturbations

Moreover, when binary signal is based on a **unique waveform**

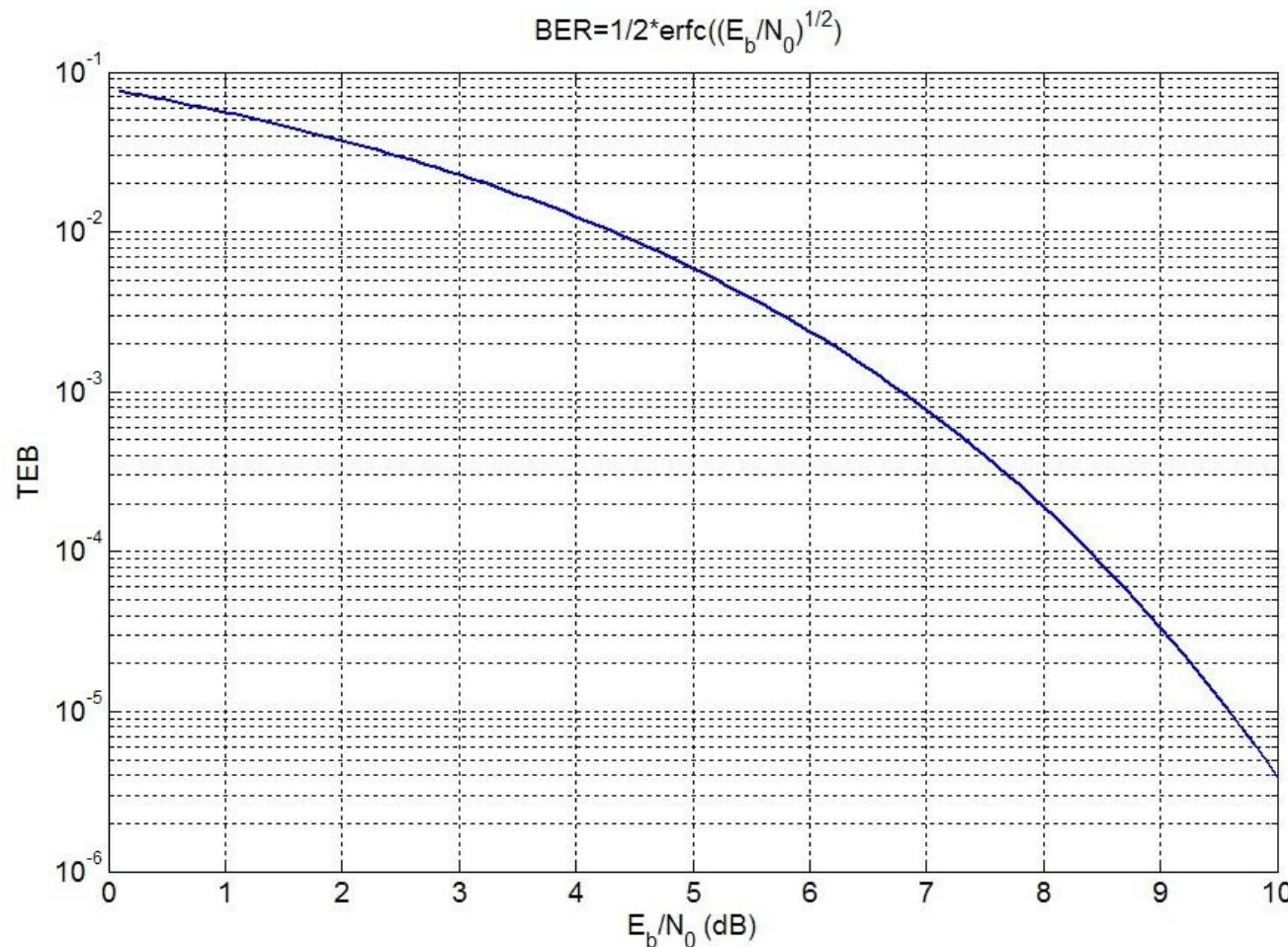


$$E_d = \int_{-\infty}^{+\infty} |h(\tau) + h(\tau)|^2 d\tau = 2 \int_{-\infty}^{+\infty} |h(\tau)|^2 d\tau = 2 \int_{-\infty}^{+\infty} |H(f)|^2 df = 4E_b$$

Thus

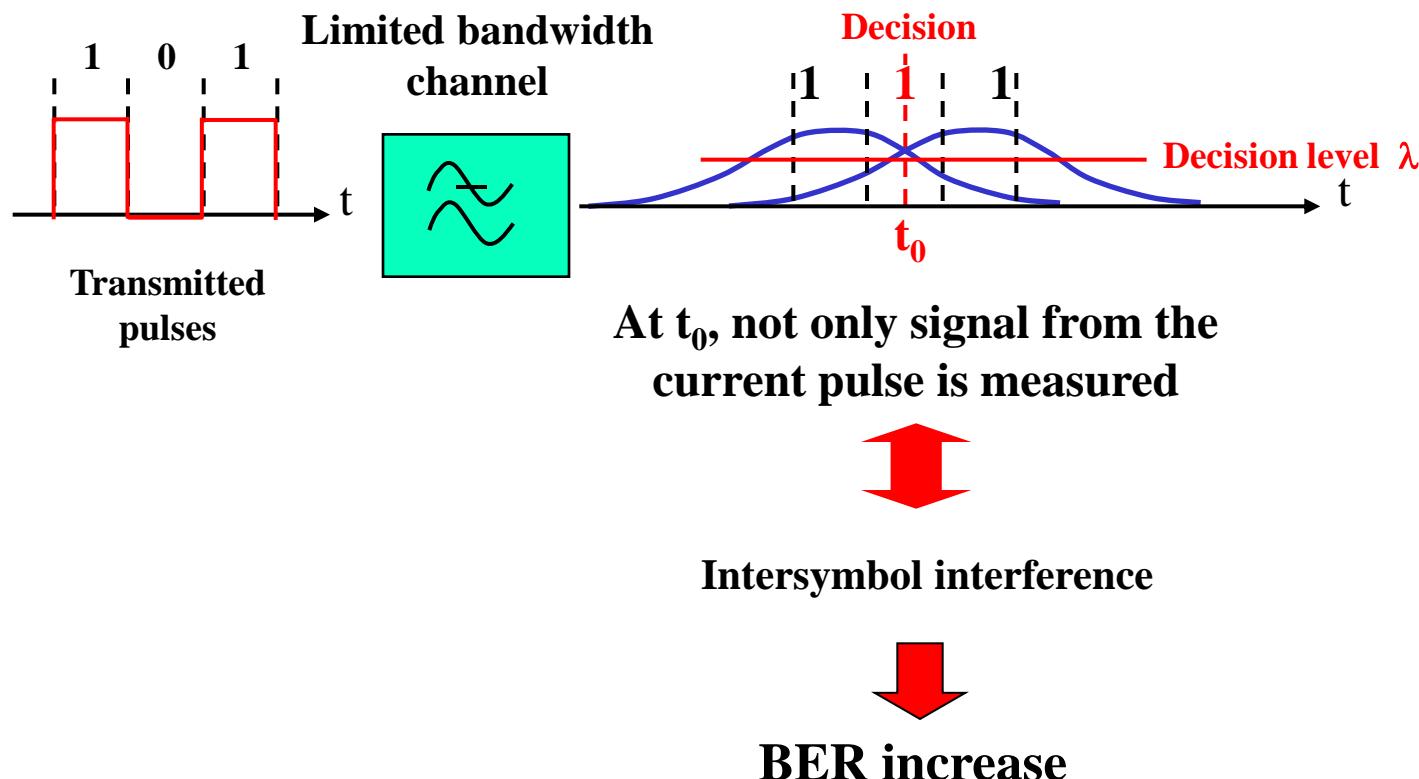
$$\Rightarrow P_{eb} = \frac{1}{2} erfc \left(\sqrt{\frac{E_b}{N_0}} \right)$$

- ← Equiproability
- ← Optimal decision level
- ← Unique waveform with $a_i = \pm 1$



- Perturbations due to the bandwidth of the transmission channel

Let us consider the consequences of the filtering of the channel (limited bandwidth) onto the received symbols.

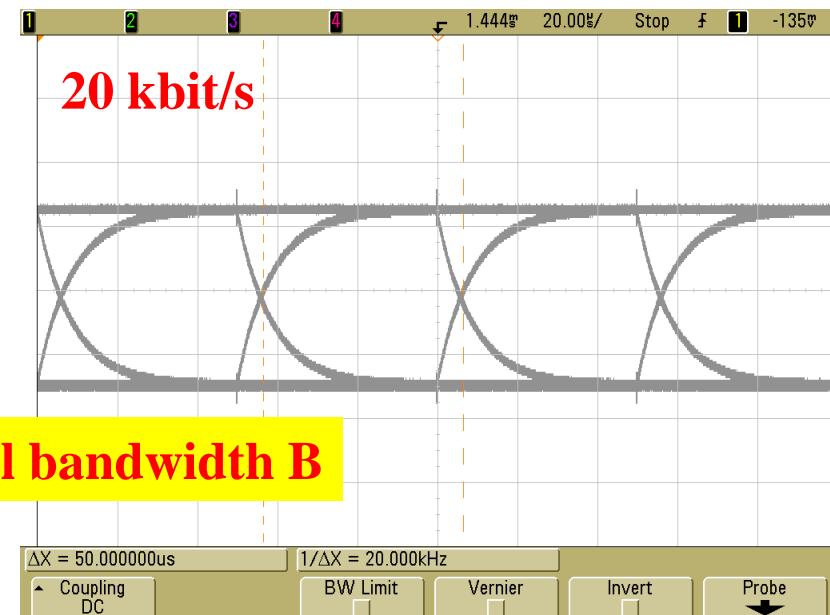
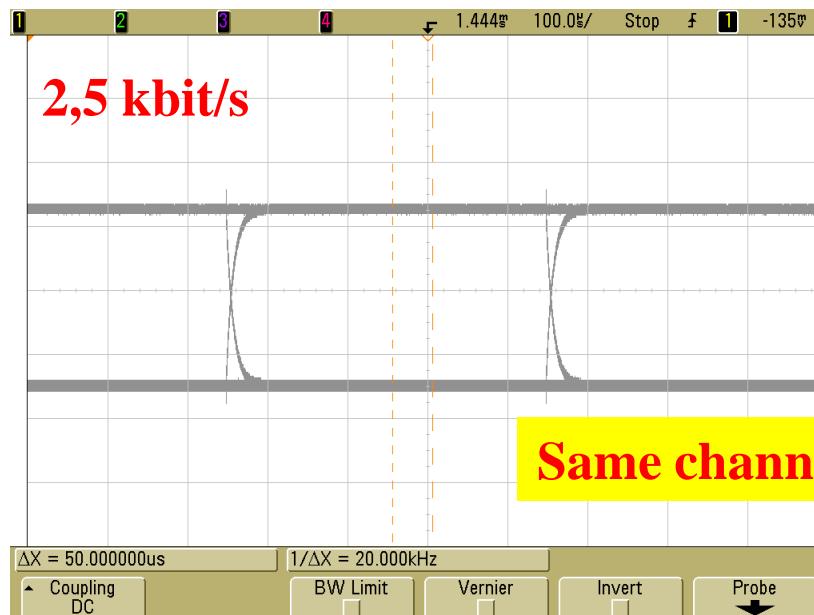


- Perturbations due to the bandwidth limitation of the transmission channel

Eye diagram :

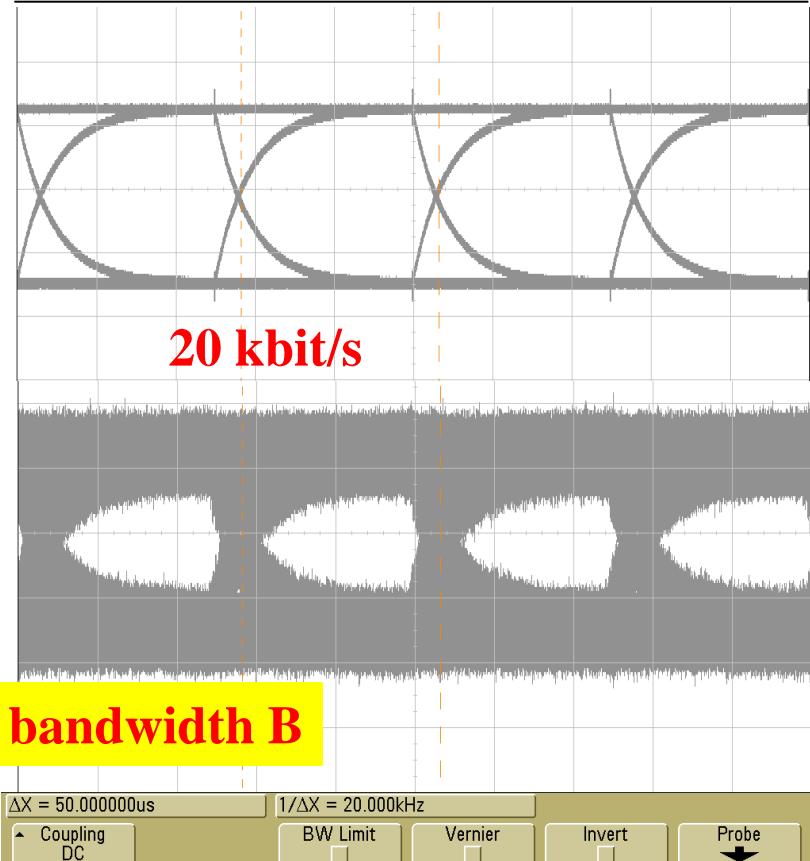
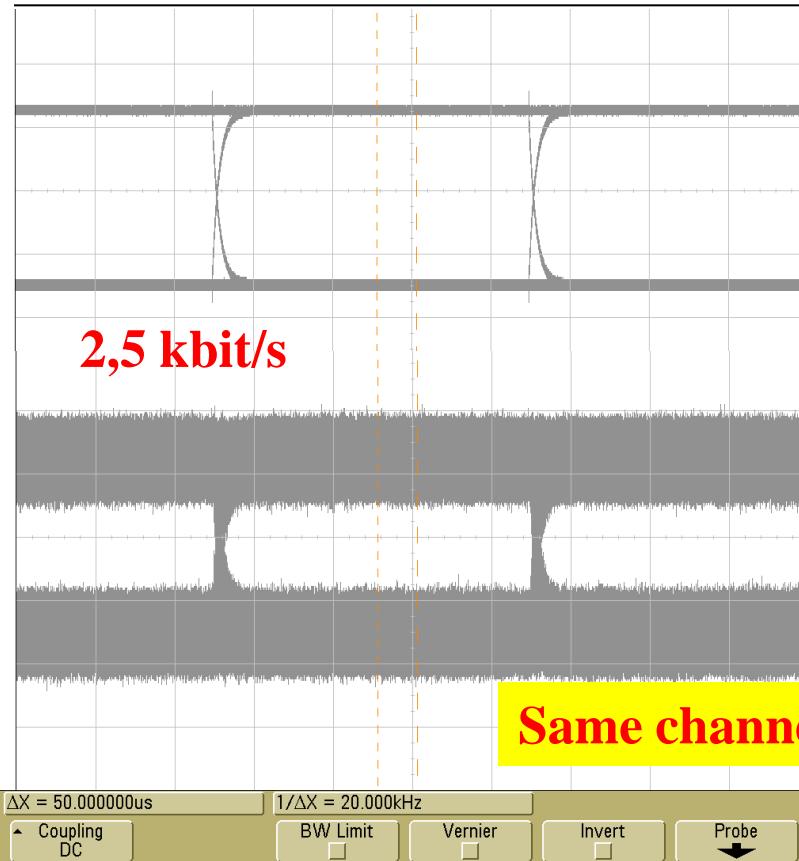
The measurement with an oscilloscope of the received symbols (∞) is called the eye diagram.

In a channel of given bandwidth **B**, when the bit rate increases the intersymbol interference increases and as a consequence, the eye pattern closes itself.

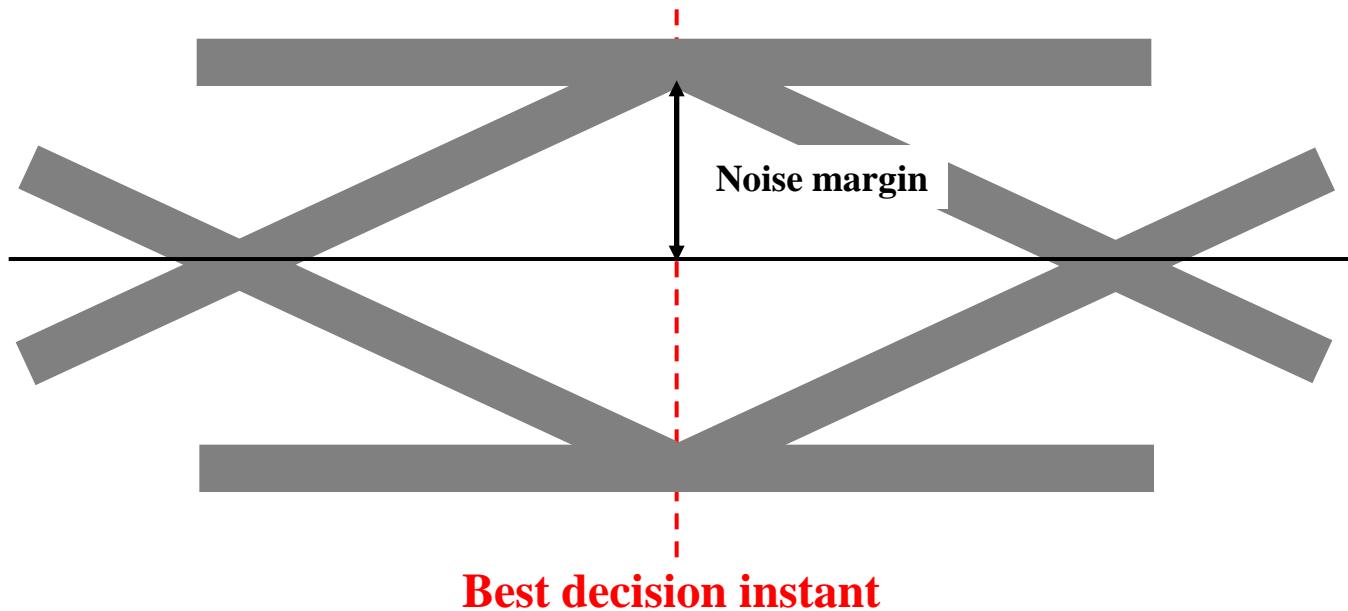


Same channel bandwidth **B**

In presence of noise the eye diagram will close itself :

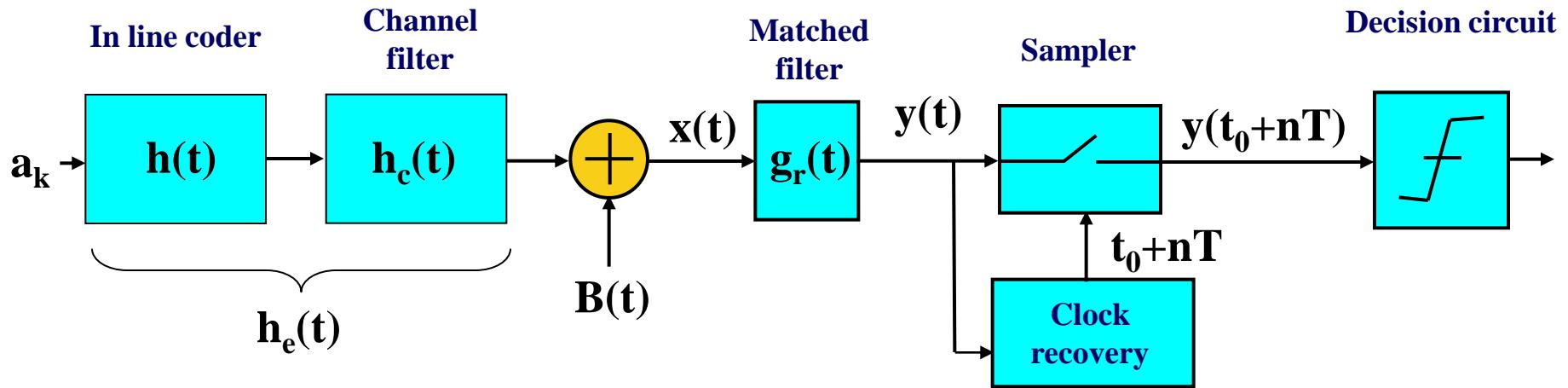


Same channel bandwidth B



The ratio between the width of the diagram lines and its vertical opening is proportional to the signal to noise ratio and is a qualitative indication of the BER and transmission quality.

- Sampled $y(t_0+nT)$ signal at the receiver



$$x(t) = \sum_k a_k h(t - kT) * h_c(t) + B(t)$$

$$y(t) = \sum_k a_k h(t - kT) * h_c(t) * g_r(t) + B(t) * g_r(t)$$

$$y(t) = \sum_k a_k r(t - kT) + b(t)$$

Where $\begin{cases} b(t) = B(t) * g_r(t) \\ r(t) = h(t) * h_c(t) * g_r(t) \end{cases}$

One samples at $t_0 + nT$

$$y(t_0 + nT) = \sum_k a_k r(t_0 + (n-k)T) + b(t_0 + nT) \quad \text{Let } m=n-k$$

$$y(t_0 + nT) = a_n r(t_0) + \sum_{m \neq 0} a_{n-m} r(t_0 + mT) + b(t_0 + mT)$$

Useful information
corresponding to
the n^{th} symbol
that we want to
recognize

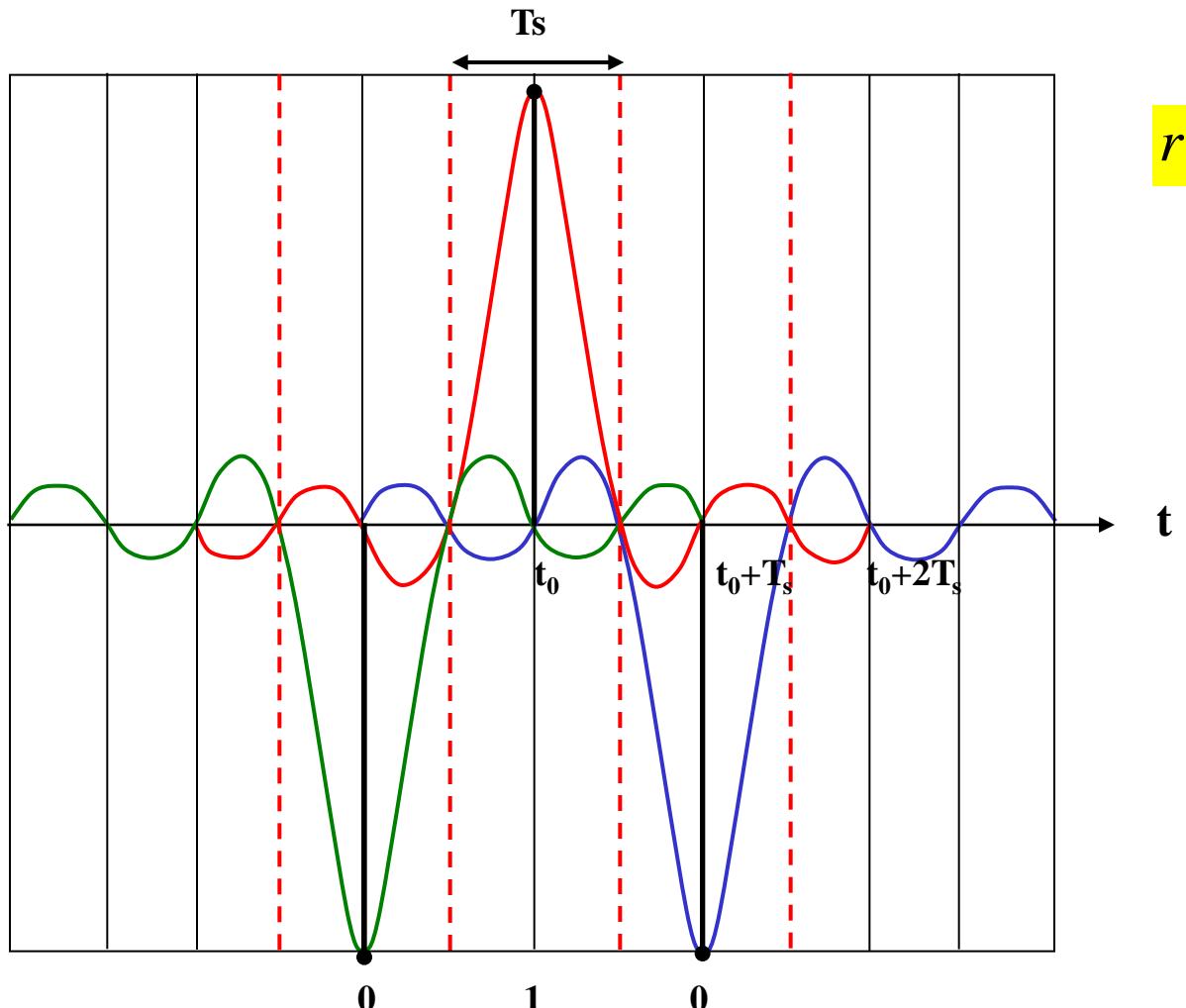
Intersymbol
interference on the
m symbols
 $m \neq n$

noise

At t_0 one wants no intersymbol interference

Mathematically this means $r(t_0 + mT) \forall m = n - k \neq 0$

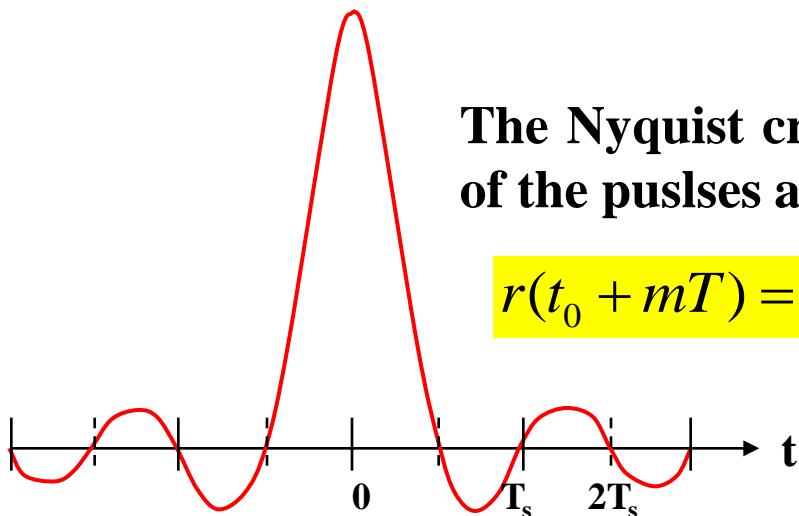
- Nyquist criterion on the time form of the received pulses**



Nyquist criterion

$$r(t_0 + mT) = 0 \quad \forall m \neq 0$$

No intersymbol
interference at the
décision times



The Nyquist criterion is a requirement that the expression of the pulses as a function of the time variable must fulfill.

$$r(t_0 + mT) = 0 \quad \forall m \neq 0$$

- Nyquist criterion on the pulses spectrum

What condition does the pulses spectrum have to fulfill in order to have no intersymbol interference at the decision time ?

Let us assume a received impulse $r(t)$ (it is assumed that there is no noise) ;

Sampled impulse $r_e(t)$

$$r_e(t) = r(t) \sum_n \delta(t - t_0 - nT)$$

$$r_e(t) = r(t) \left(\sum_n \delta(t - nT) \right) * \delta(t - t_0)$$

$$R_e(f) = R(f) * \left(\frac{1}{T} \sum_n \delta(f - \frac{n}{T}) \right) e^{-j2\pi f t_0}$$

$$R_e(f) = R(f) * \left(\frac{1}{T} \sum_n \delta(f - \frac{n}{T}) e^{-j2\pi f t_0} \right)$$

$$R_e(f) = \frac{1}{T} \sum_n R(f - \frac{n}{T}) e^{-j2\pi \frac{nt_0}{T}} \quad (1)$$

Moreover $r_e(t) = r(t) \sum_n \delta(t - t_0 - nT)$

$$r_e(t) = \sum_n \underline{r(t_0 + nT)} \delta(t - t_0 - nT)$$

Independent of t

$$R_e(f) = \sum_n r(t_0 + nT) e^{-j2\pi f(t_0 + nT)} \quad (2)$$

$$(1)=(2) \Rightarrow \frac{1}{T} \sum_n R(f - \frac{n}{T}) e^{-j2\pi \frac{nt_0}{T}} = \sum_n r(t_0 + nT) e^{-j2\pi f(t_0 + nT)} \quad (3)$$

Moreover, the Nyquist condition $r(t_0 + mT) = 0 \quad \forall m \neq 0$

$$\Rightarrow \frac{1}{T} \sum_n R(f - \frac{n}{T}) e^{-j2\pi \frac{nt_0}{T}} = r(t_0) e^{-j2\pi ft_0}$$

$$\Rightarrow \sum_n R(f - \frac{n}{T}) e^{-j2\pi (f - \frac{n}{T}) t_0} = Tr(t_0)$$

$$\Rightarrow \sum_n \frac{R(f - \frac{n}{T})}{r(t_0)} e^{-j2\pi (f - \frac{n}{T}) t_0} = T$$

Nyquist criterion : if $R(f)$ satisfies this equation then there is no ISI at the decision time

$$\Leftrightarrow \sum_n R^{(t_0)}\left(f - \frac{n}{T}\right) = T$$

with $R^{(t_0)}(f) = \frac{R(f - \frac{n}{T})}{r(t_0)} e^{j2\pi f t_0}$

}

FT of the impulse normalized
with regards to $r(t_0)$ and phase
shifted ...

It is the summ of the translates spectra.

If the spectrum width $B < 1/2T$, then the summ of the translated spectra cannot be constant ($= T$).

One cannot transmitt without ISI a signal of modulation speed $R = \frac{1}{T}$ in a frequency band $< \frac{1}{2T}$

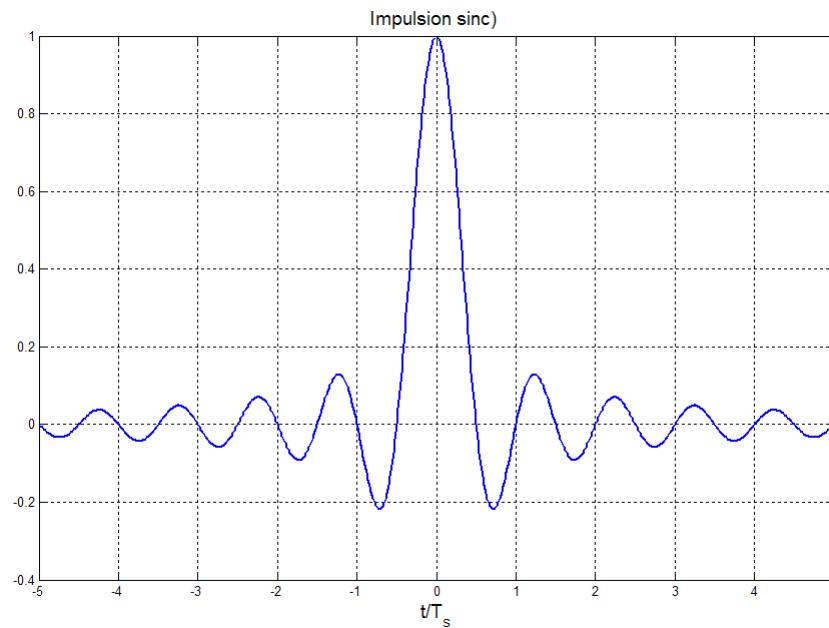
Goal : no ISI at the t_0-nT decision times (at other instants we do not care).

- (1) The Nyquist criterium as a function of time : the pulses have to be = 0 $\forall t = kT$ (T symbol time).
- (2) One converts the Nyquist criterion into a frequency criterion \Rightarrow one obtains a condition on the spectrum of the impulses.
- (3) One deduces a necessary (but not sufficient) condition so that the Nyquist criterion be respected :

$$B \geq f/2 \text{ (} f = \text{symbol frequency})$$

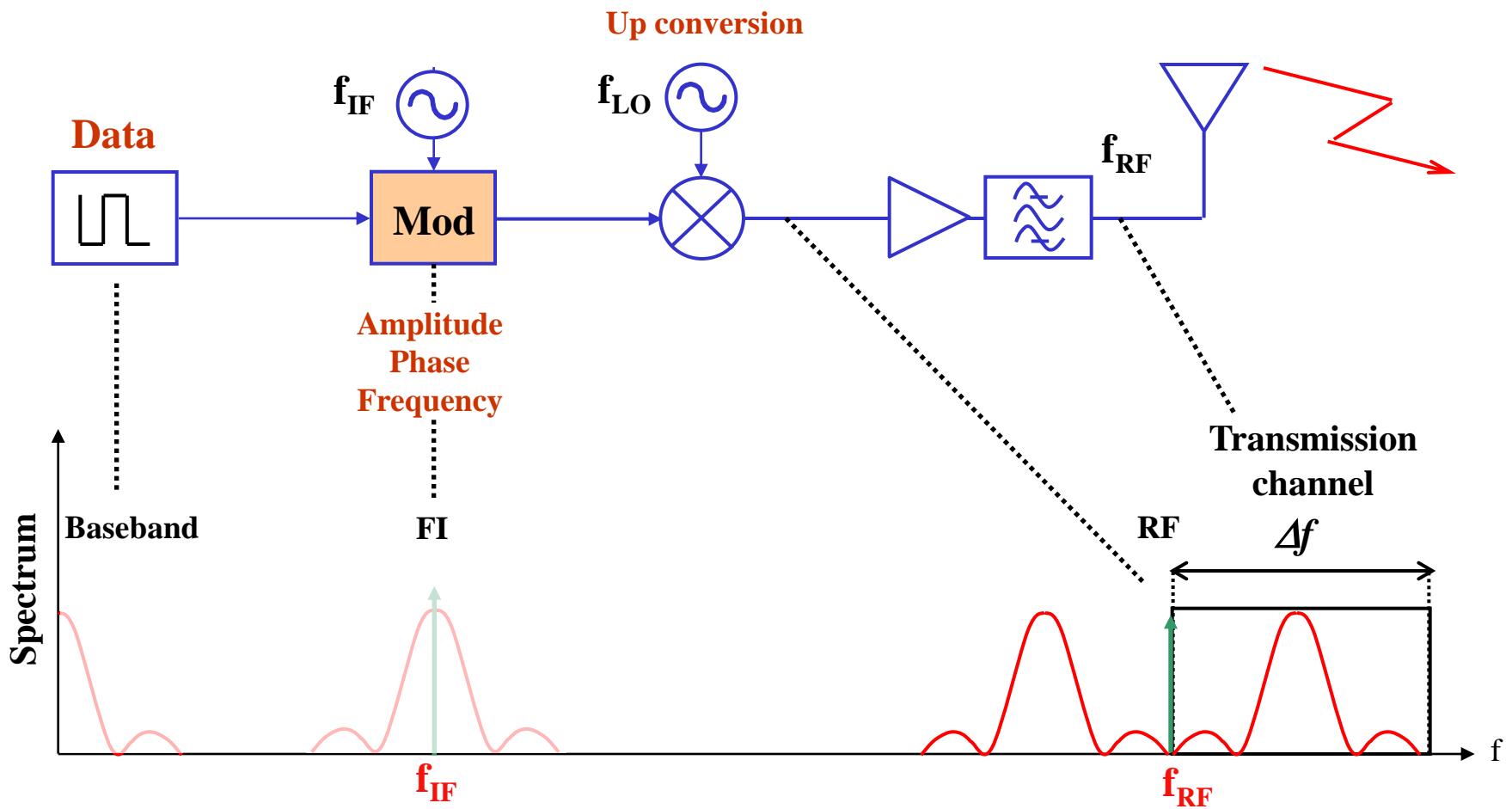
To have no ISI, one can choose any impulse that satisfies the Nyquist criterion (cancellation every kT) : sinus cardinal rised cosine ...

- Exemple of impulse that satisfies the Nyquist criterion :



Transmission over a carrier freq Matching of the signal to the channel

- Digital transmission over a carrier frequency :



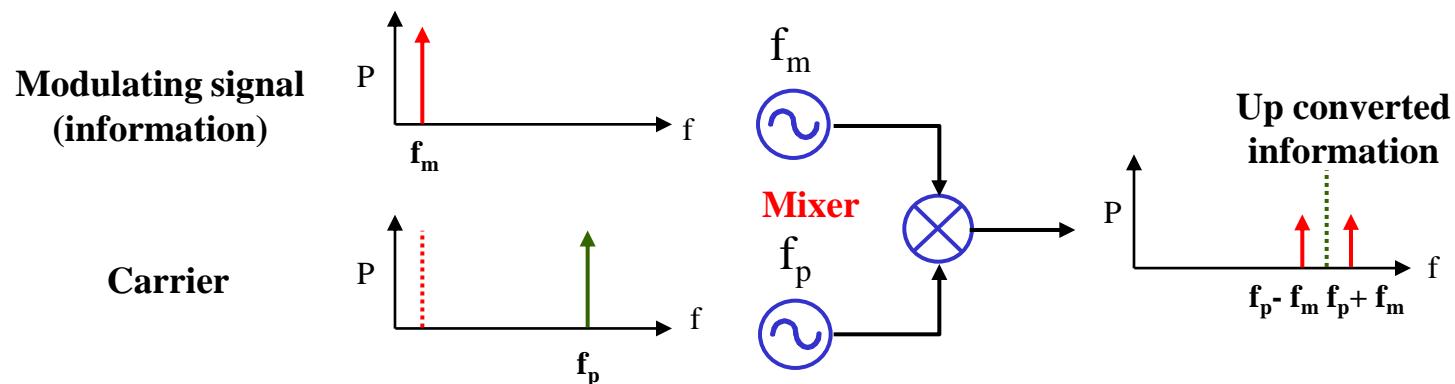
Goal : matching the transmitted signal to the channel

Basic principle → *the product*

$$\cos a * \cos b = \frac{1}{2} [\cos(a+b) + \cos(a-b)]$$

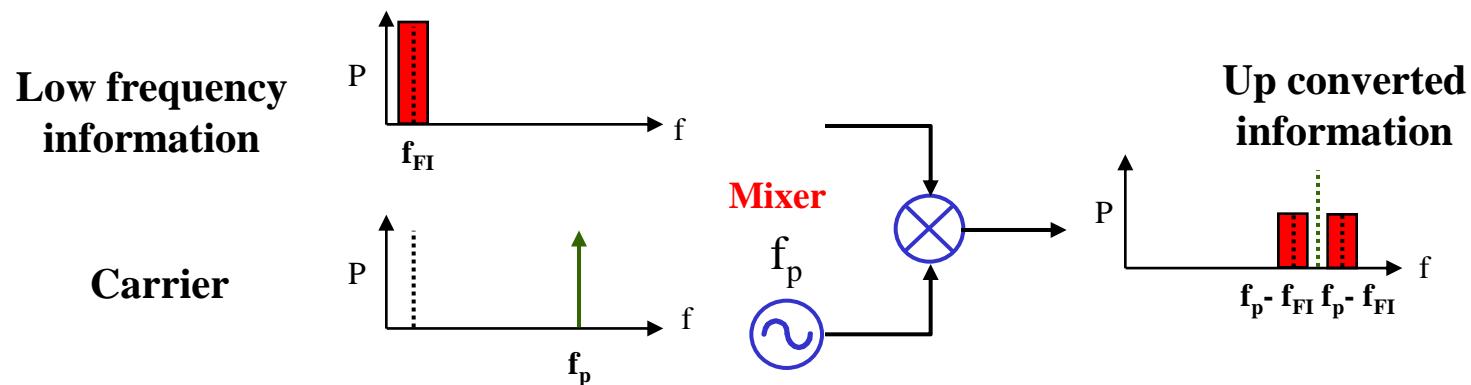
The simple fact of **multiplying** a signal of frequency f_m by a signal of frequency f_p generates two signals : one at $f_p - f_m$ and the other at $f_p + f_m$.

$$A \cos(2\pi f_p t) * B \cos(2\pi f_m t) = \frac{1}{2} AB \left\{ \cos[2\pi(f_p + f_m)t] + \cos[2\pi(f_p - f_m)t] \right\}$$



This may be considered as a **transposition** of the low frequency signal **into an HF band**.

In the same manner, the product of a signal that occupies a frequency band B by an f_p carrier frequency results in the up conversion of the considered band from both sides of f_p .

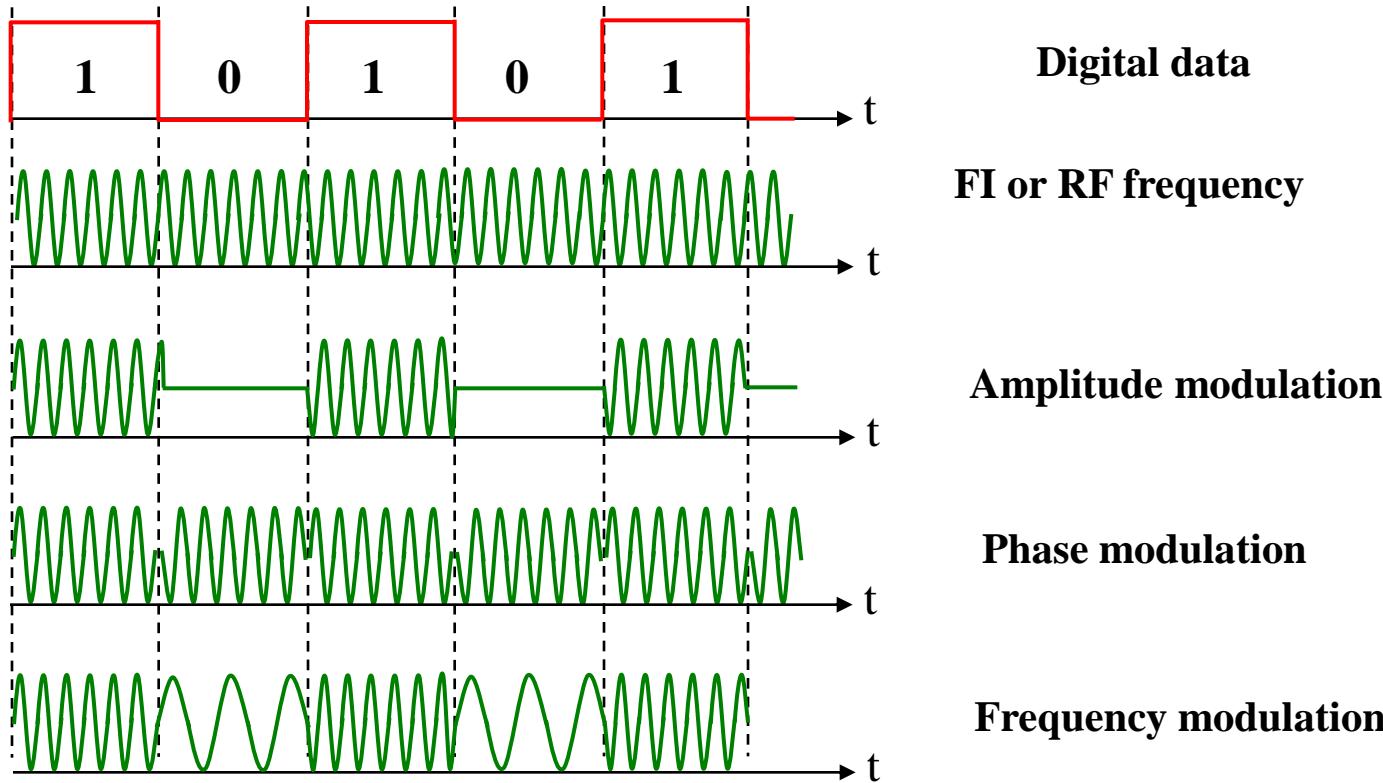


Typically, the spectrum that is up converted corresponds to an intermediate frequency (IF) that has undergone a **digital modulation**.

The **digital modulation** of a carrier frequency (or of an IF) consists in **varying the amplitude, the frequency or the phase of sinusoidal wave (carrier)** as a function of the binary data to transmitt.

The binary data may be gathered in ensembles of M digits ($M=2^n$), in that case **the modulated signal (carrier) may have $M \neq$ states**.

- Illustration : binary digital modulations



Le choice of the modulation format will be made according to two criteria :

- ➔ Spectral efficiency (bit/s)/Hz
- ➔ the BER for a same S/N

Technical criteria (simplicity of circuits...) may also be taken into account.

For each digital modulation format it is important to determine the :

- ➔ corresponding spectrum
- ➔ corresponding BER(S/N)
- ➔ The modulation and demodulation circuits

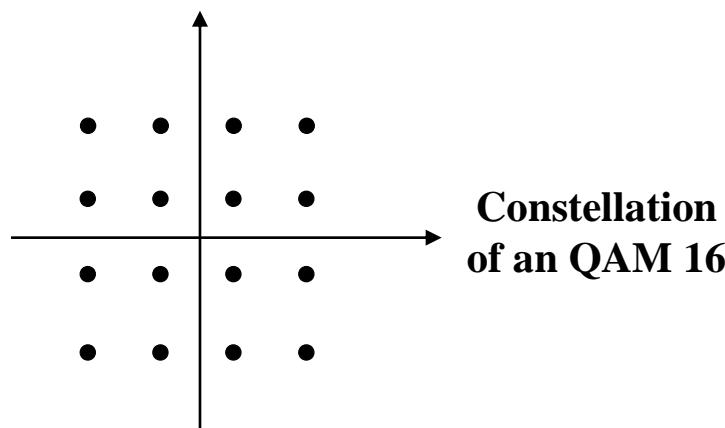
One may represent the different modulation states in the Fresnel plane

For the **digital modulations of amplitude and of phase** one can represent the different states of the modulated signal $s(t)$ in the **Fresnel plane**.

$$s(t) = A_{k(i)} \cos(2\pi f_0 t + \phi_{k(i)}) \quad \text{avec} \quad i \in \{0, \dots, M-1\} \text{ où } M = 2^n$$

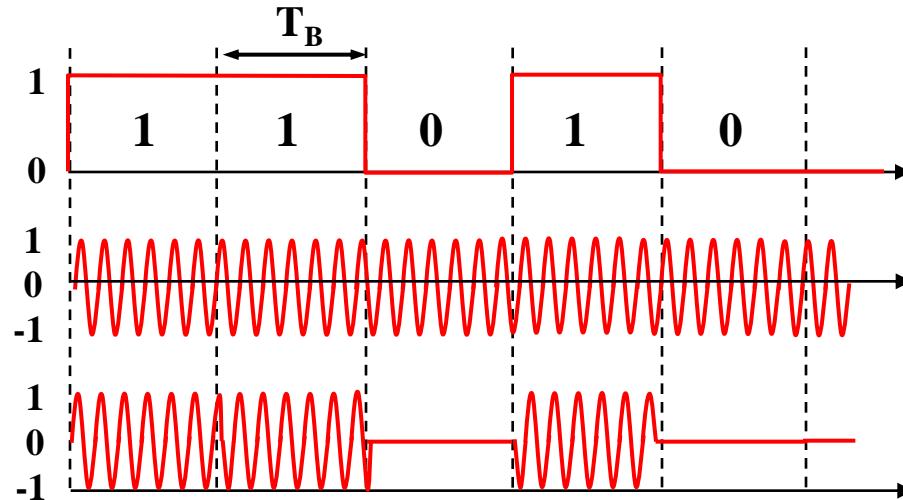
Each modulation state i is represented by vector of module A_i and of angle ϕ_i .

This representation of the states of modulation is called a **constellation**.



- OOK or ASK Modulation :

OOK : On Off Keying
 ASK : Amplitude Shift Keying



The modulated signal $s(t)$ is the product of the $i(t)$ baseband data (“i” for info) by the carrier $p(t)$.

$$\left. \begin{array}{l} i(t) = \sum_k a_k h(t - kT) \\ p(t) = A \cos 2\pi f_0 t \end{array} \right\} \Rightarrow s(t) = p(t) \cdot i(t) = A \cos(2\pi f_0 t) \cdot \sum_k a_k h(t - kT)$$

Spectre d'une modulation OOK :

- Bilateral PSD of the baseband signal

$$\gamma_i = \frac{T_s}{4} \sin c^2(fT_s) + \frac{1}{4} \delta(f)$$

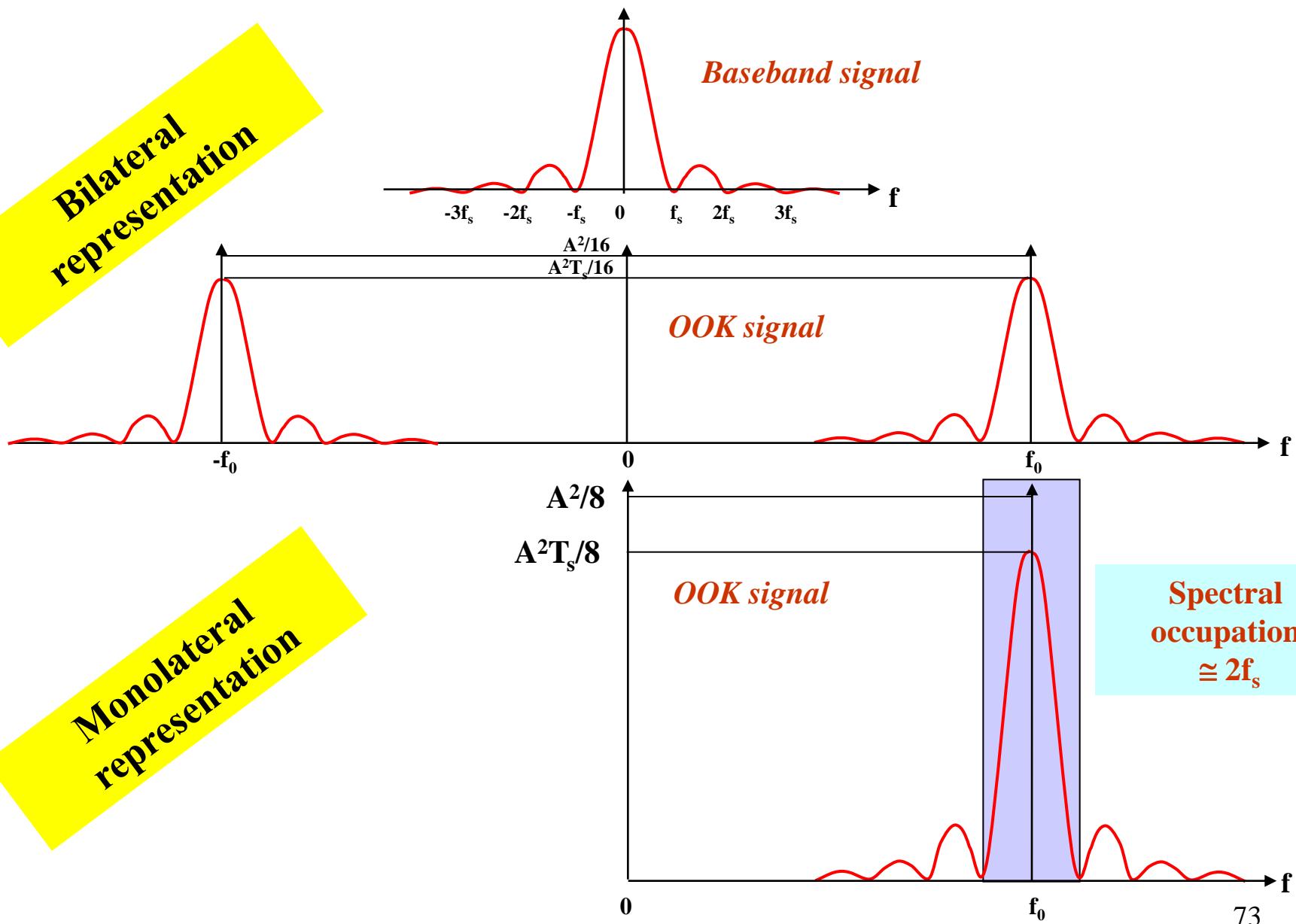

Average signal

- Bilateral PSD γ_s of the OOK signal *One admits the result that is not immediate and requiers to use the complex envelope notion...*

$$\begin{aligned} \gamma_s &= \frac{A^2}{16} [\delta(f - f_0) + \delta(f + f_0)] \\ &+ \frac{A^2 T_s}{16} [\sin c^2 [(f - f_0)T_s] + \sin c^2 [(f + f_0)T_s]] \end{aligned}$$

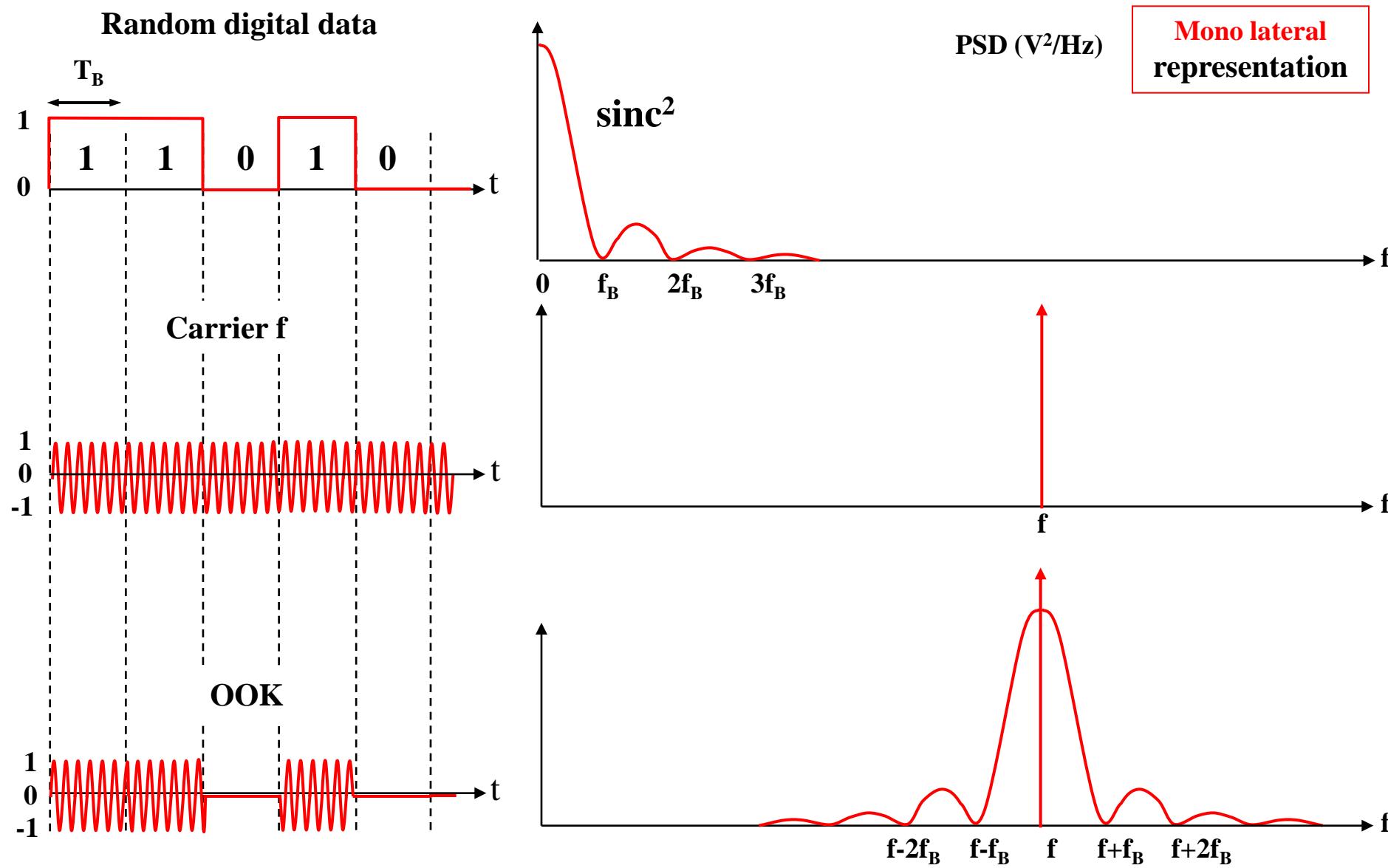
Transmission over a carrier frequency

ASK modulation



Transmission over a carrier frequency

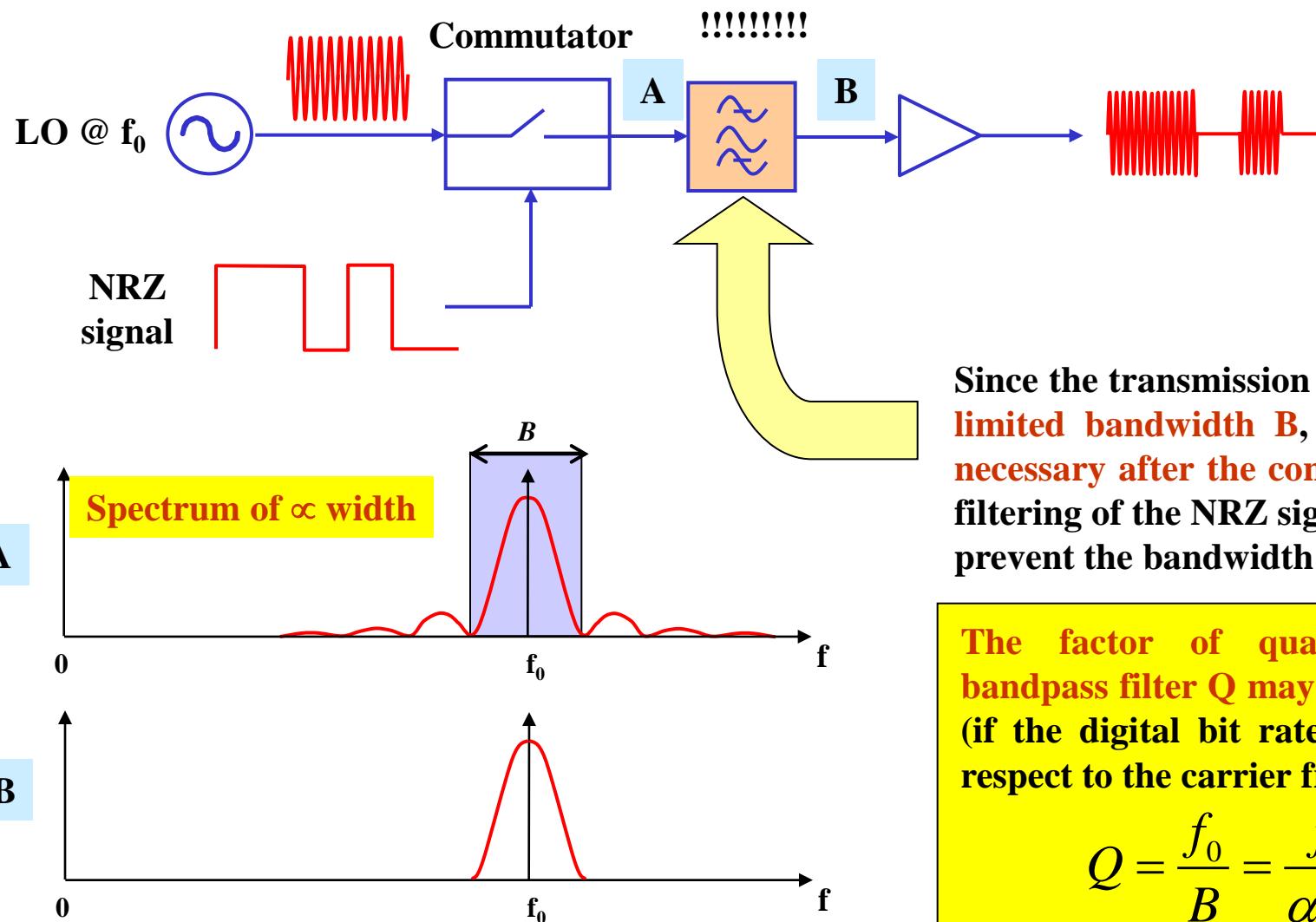
ASK modulation



Transmission over a carrier frequency

ASK modulation

ASK modulator:



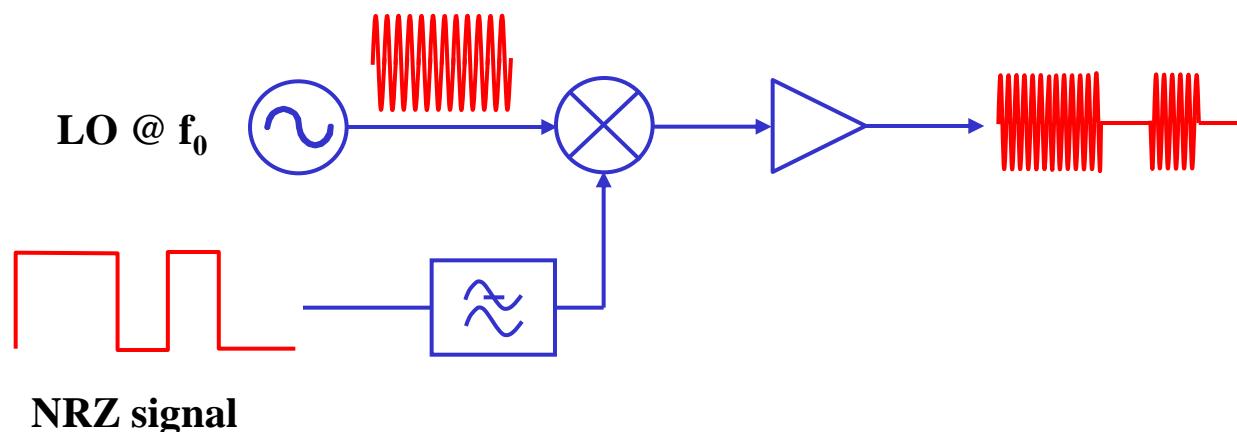
Since the transmission channel has a limited bandwidth **B**, a filtering is necessary after the commutator (the filtering of the NRZ signal would not prevent the bandwidth to be ∞ in A)

The factor of quality of the bandpass filter **Q** may be very high (if the digital bit rate is low with respect to the carrier frequency) :

$$Q = \frac{f_0}{B} = \frac{f_0}{\alpha f_b}$$

ASK modulator:

To avoid this problem the ASK modulator may be built as follows :

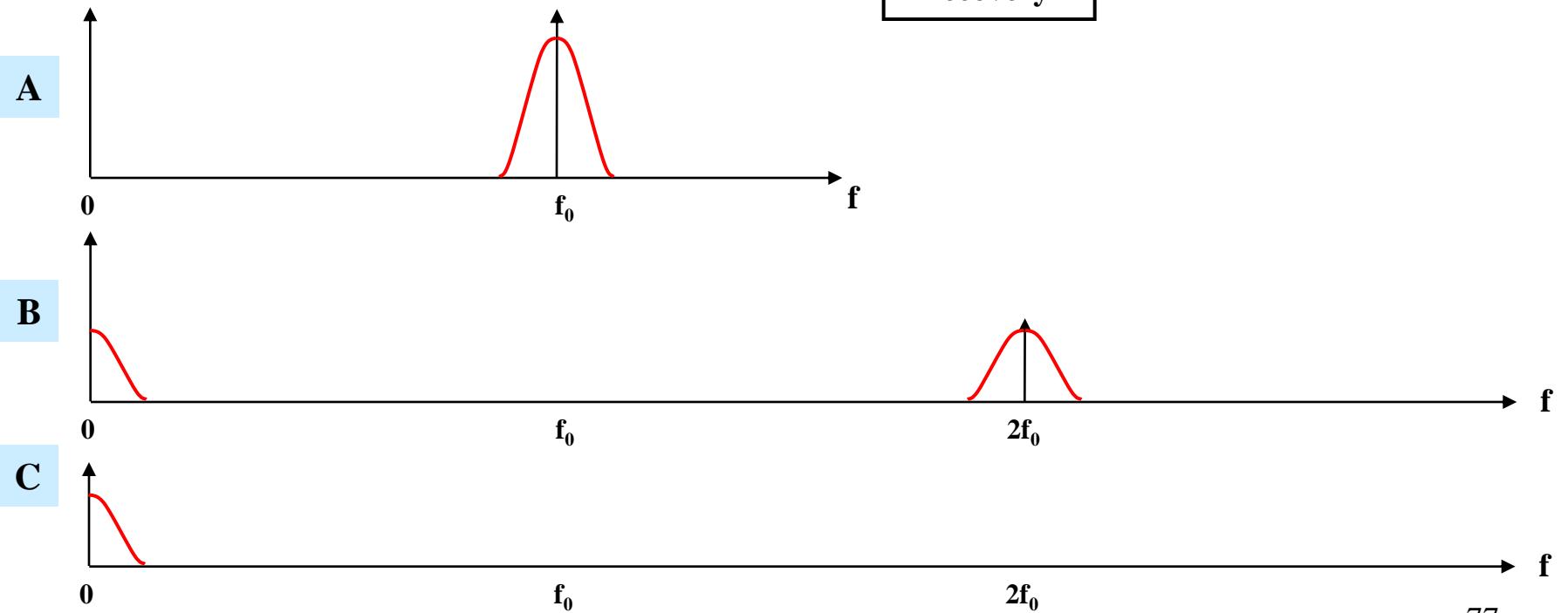
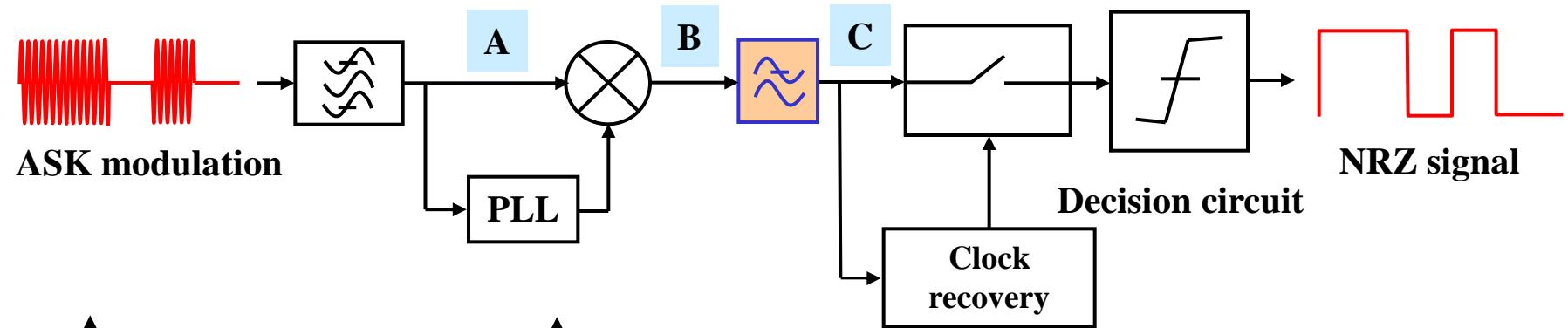


Transmission over a carrier frequency

ASK modulation

ASK demodulator :

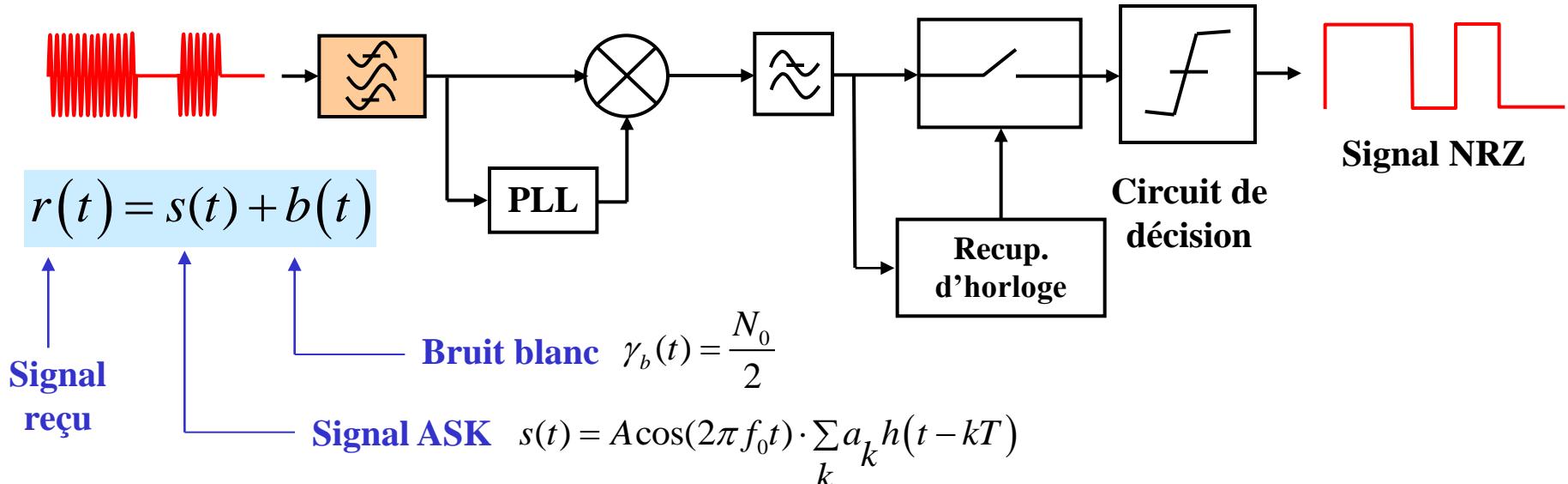
- Lock-in amplification



Transmission over a carrier frequency

ASK modulation

- BER



Si le **filtre passe bande** est **adapté** (aux impulsions du signal ASK) :

$$P_{eb} = \frac{1}{2} erfc \left(\sqrt{\frac{E_d}{4N_0}} \right)$$

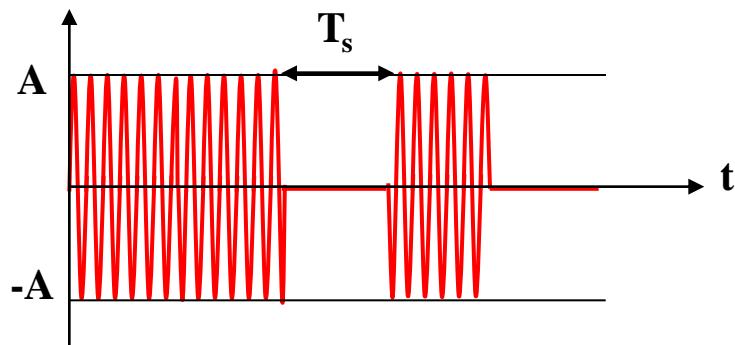
Vu à la télé

Même résultat
qu'en bande de base

Équiprobabilité 0 & 1 ; Seuil optimal

Où E_d est l' ϵ du *signal différence* entre les symboles $E_d = \int |S_1(t) - S_0(t)|^2 dt$

$$P_{eb} = \frac{1}{2} \operatorname{erfc} \left(\sqrt{\frac{E_d}{4N_0}} \right) \quad E_d = S_1(t) - S_0(t) \quad \varepsilon \text{ du signal différence entre les symboles}$$



$$E_d = \int_0^{T_s} [A \cos \omega_0 t - 0]^2 dt = \int_0^{T_s} A^2 \cos^2(\omega_0 t) dt$$

Or, l' ε moyenne par bit est $E_b = \frac{1}{2} E_d$

$$\Rightarrow P_{eb} = \frac{1}{2} \operatorname{erfc} \left(\sqrt{\frac{E_b}{2N_0}} \right)$$

$$* \quad E_b = \frac{1}{2} \int_0^{T_s} S_1^2(t) dt + \frac{1}{2} \int_0^{T_s} S_0^2(t) dt = \frac{1}{2} \int_0^{T_s} S_1^2(t) dt = \frac{1}{2} \int_0^{T_s} A^2 \cos^2(\omega_0 t) dt = \frac{1}{2} E_d$$

$$p_1 = p_0 = 1/2$$

Sachant que $E_d = \int_0^{T_s} [A \cos \omega_0 t - 0]^2 dt = \frac{A^2 T_s}{2}$ $\Rightarrow P_{eb} = \frac{1}{2} erfc\left(\sqrt{\frac{A^2 T_s}{8 N_0}}\right)$

$$E_d = \int_0^{T_s} A^2 \cos^2(\omega_0 t) dt = A^2 \int_0^{T_s} \frac{\cos(2\omega_0 t) + 1}{2} dt = A^2 \int_0^{T_s} \frac{\cos(2\omega_0 t) + 1}{4\omega_0} d(2\omega_0 t)$$

$$E_d = \frac{A^2}{4\omega_0} [\sin(2\omega_0 t) + 2\omega_0 t]_{t=0}^{t=T_s} = \frac{A^2 T_s}{2}$$

Rque : dans le cas particulier traité pour l'étude en bande de base on trouvait $P_{eb} = \frac{1}{2} erfc\left(\sqrt{\frac{E_b}{N_0}}\right)$ ≠
car on avait alors $a_1=+1$ et $a_0=-1$.

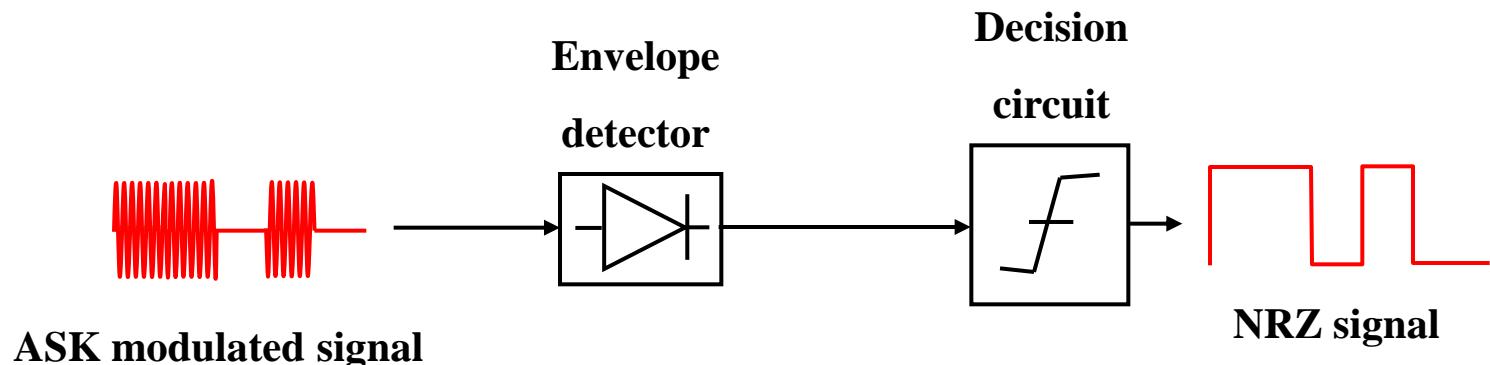
En modulation ASK on a $a_1=1$ et $a_0=0$.

Ceci explique le facteur 2 au dénominateur... $P_{eb} = \frac{1}{2} erfc\left(\sqrt{\frac{E_b}{2N_0}}\right)$ ≠

En effet ; en moyenne dans ce cas l'ε du signal est deux fois moindre...

ASK demodulator:

- Démodulation by envelope detection



• BER

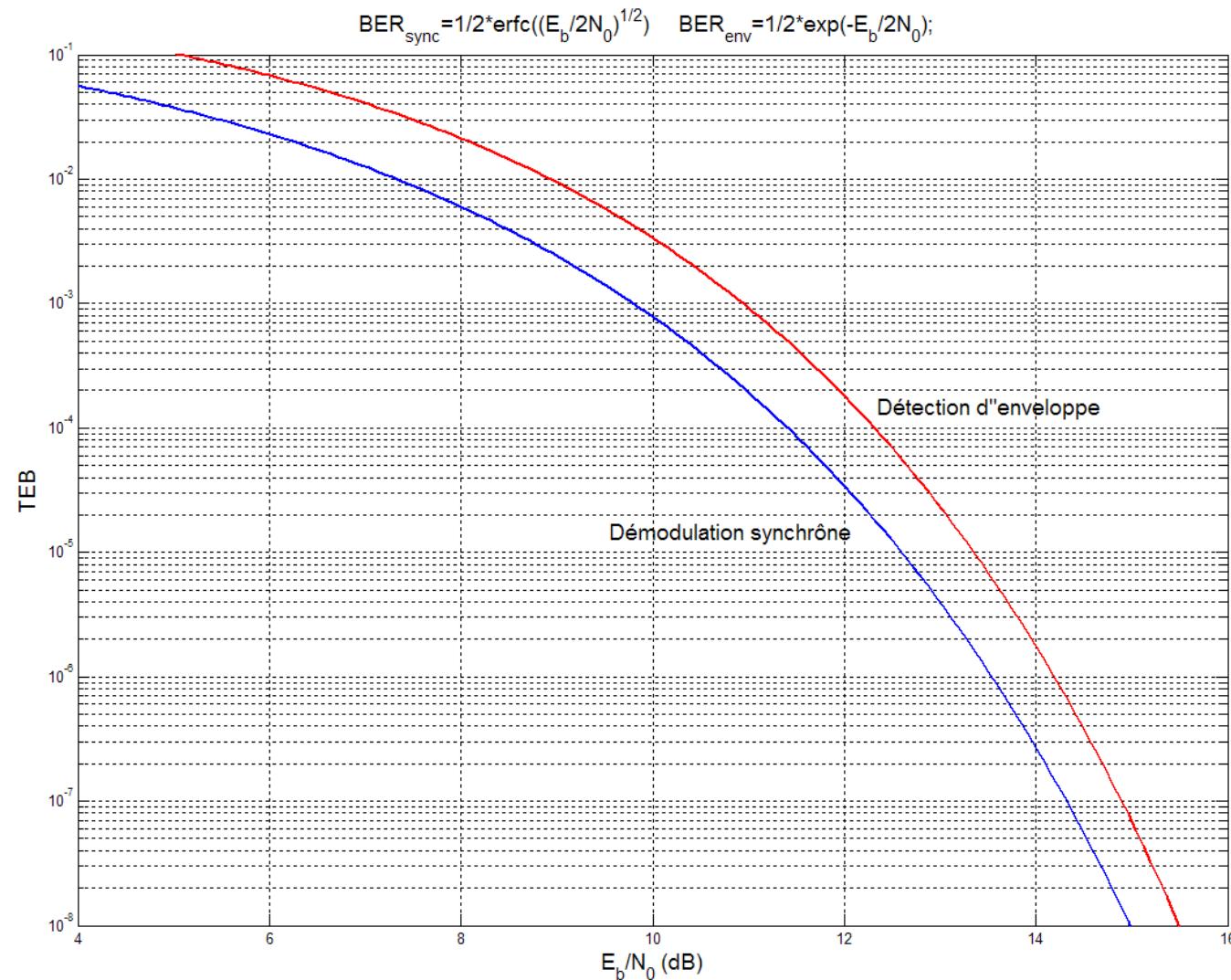
“One” shows that

$$P_{eb} = \frac{1}{2} e^{-\frac{E_b}{2N_0}}$$

La démodulation par détection d'enveloppe requiert un circuit plus simple que la démodulation cohérente. Cependant le BER est plus élevé.

Transmission over a carrier frequency

ASK modulation



Avantages et inconvénients de l'ASK :

Avantage : Simplicité de circuits (comparée aux autres modulations numériques) et donc faible coût

Inconvénients: Moins performante que les autres modulations numériques en termes d'efficacité spectrale (bit/Hz) et de TEB pour un même S/N.

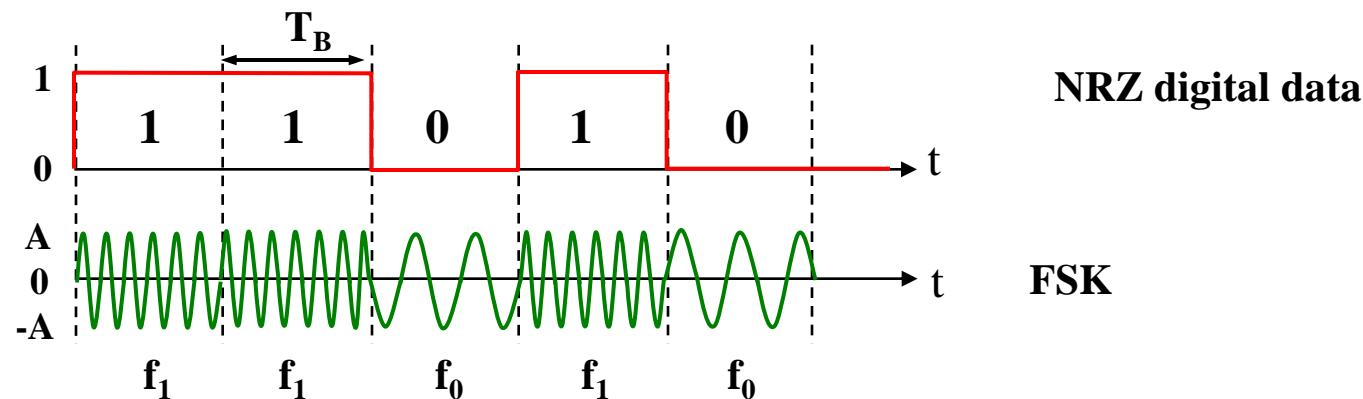
En raison de son **faible coût**, la modulation ASK est utilisée dans les **systèmes de transmission grand public courtes distances**. Deux bandes de fréquences porteuses à 224 MHz et 433 MHz sont normalisées pour ce type d'applications

Rque: L'ASK est utilisée en **télécommunications par fibre optique** car les détecteurs optiques ne sont sensibles qu'aux variations d'intensité du signal (insensibles à la fréquence et à la phase). Dans ces systèmes les fréquences porteuses sont de l'ordre de 193 THz (fréquences optiques infrarouges) et les débits peuvent atteindre 40 Gbit/s...

FSK Modulation

FSK : Frequency Shift Keying

A « 0 » is coded by the frequency $f_0 = f_p - \Delta f$ a « 1 » is coded by the frequency $f_1 = f_p + \Delta f$



An FSK modulation is characterized by:

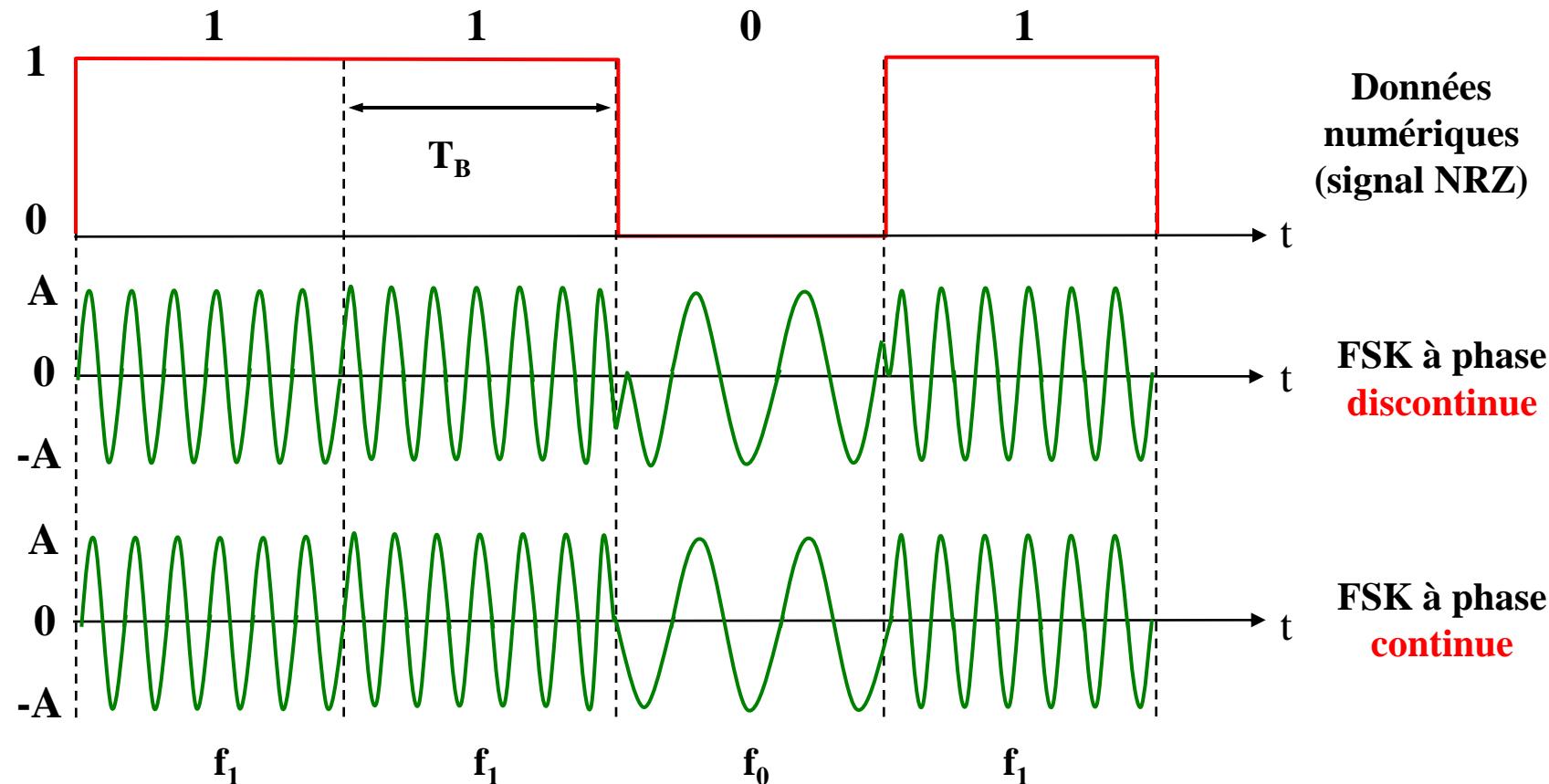
$$\left\{ \begin{array}{ll} f_p & \text{Center frequency of the FSK spectrum} \quad f_p = \frac{f_1 + f_0}{2} \\ \Delta f & \text{Frequency excursion} \quad \Delta f = \frac{|f_1 - f_0|}{2} \\ f_B & \text{Bit rate} \quad f_B = \frac{1}{T_B} \end{array} \right.$$

Transmission over a carrier frequency

FSK modulation

One distinguishes two types of FSK :

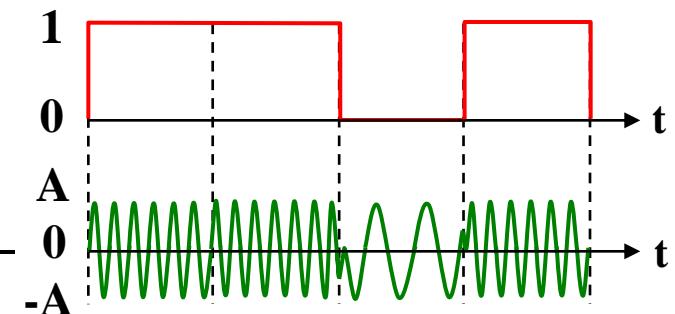
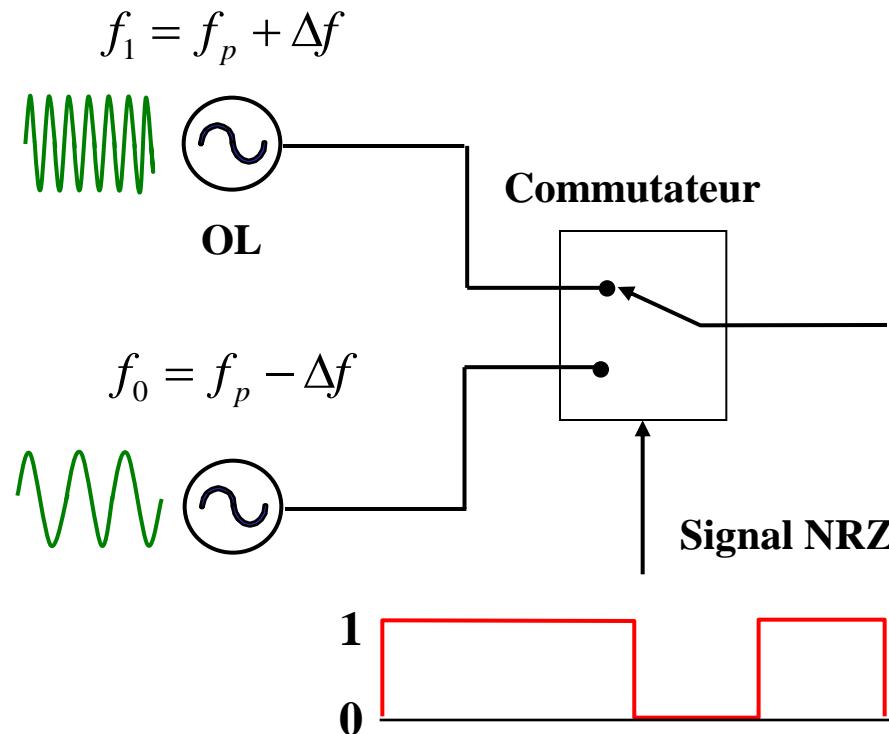
- Discontinuous phase FSK
- Continuous phase FSK



La FSK à phase discontinue présente un spectre plus large que celui d'une FSK à phase continue

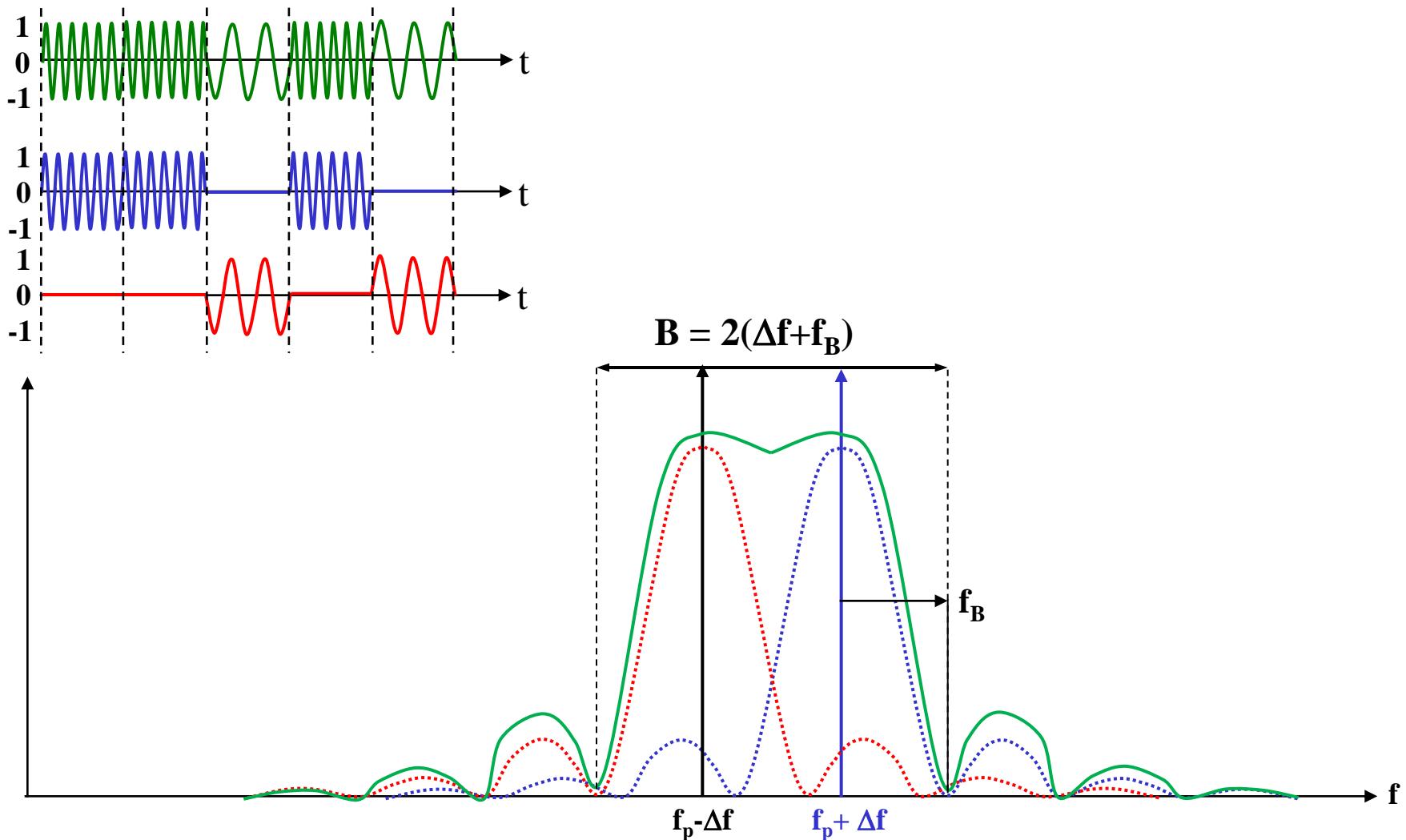
- FSK à phase discontinue

Modulateurs FSK (phase discontinue) :



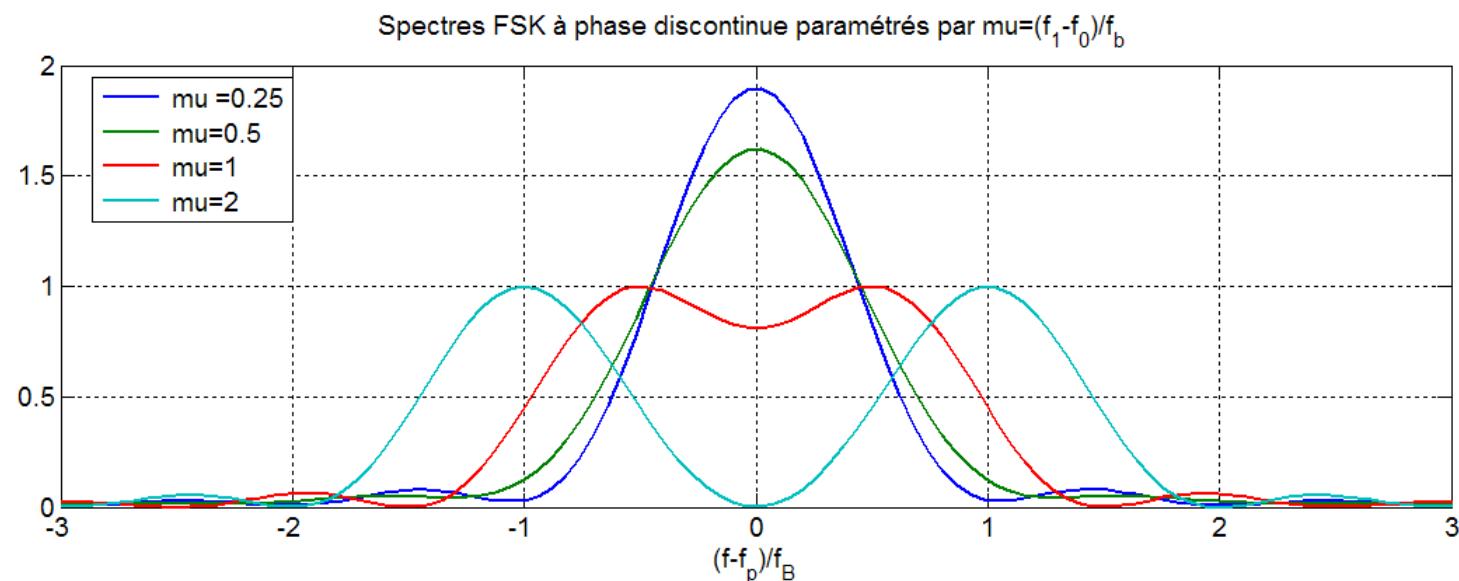
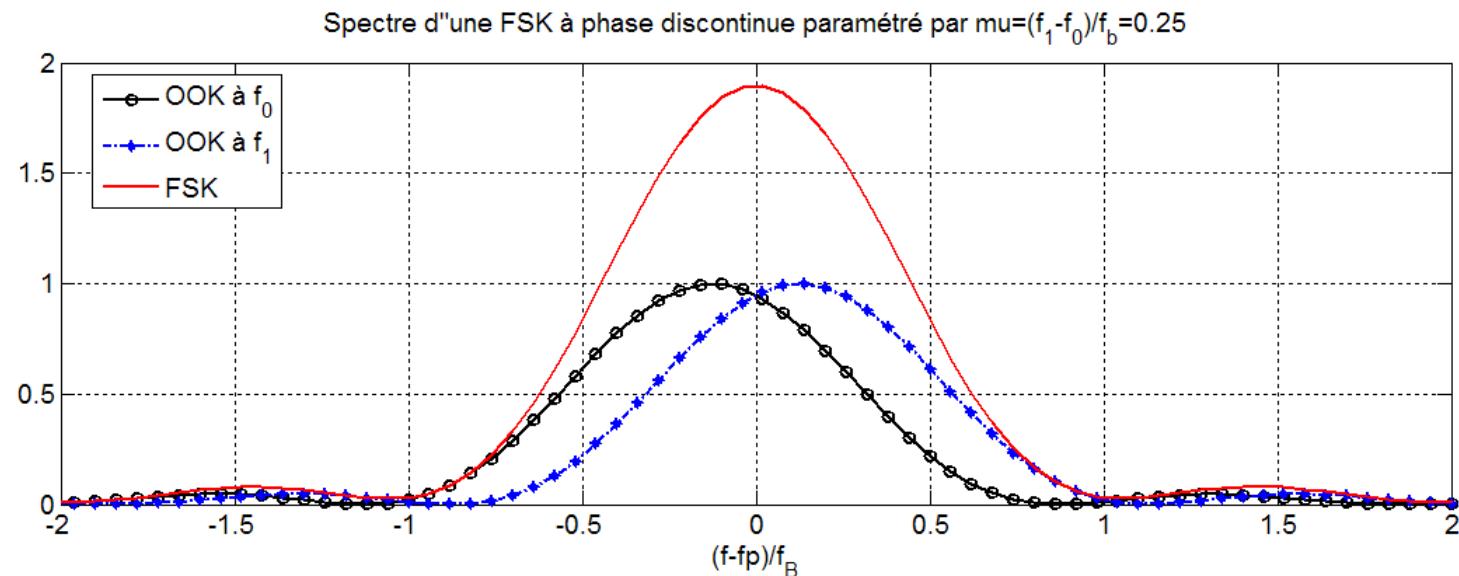
Aux instants de
commutation la phase
relative des deux OL
est qcq.

- DSP d'un signal FSK à phase discontinue \Leftrightarrow superposition de deux ASK



Transmission sur fréquence porteuse

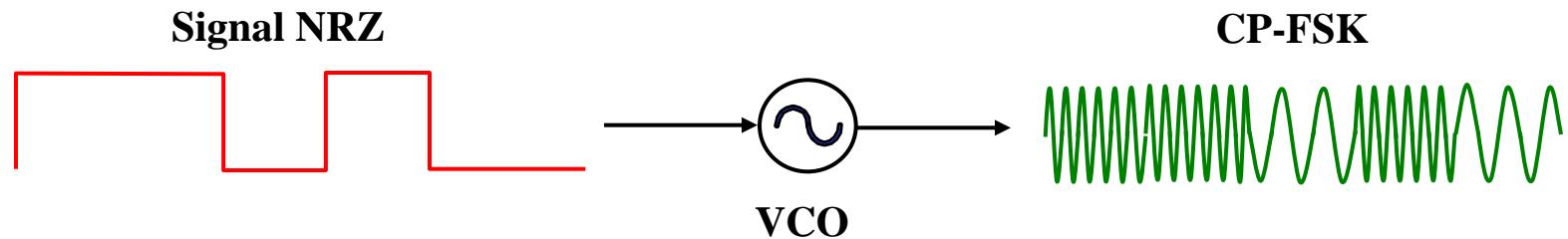
Modulation FSK



En pratique la FSK est peu utilisée, on lui préfère la **CP-FSK**
 (Continuous Phase Frequency Shift Keying)

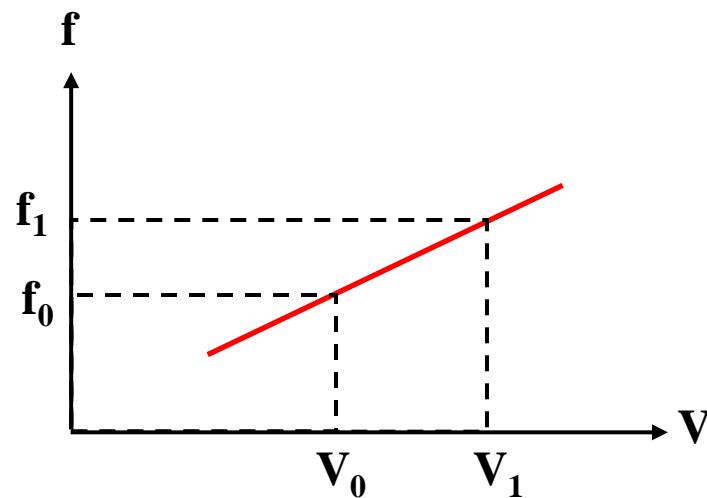
- **FSK à phase continue ou CP-FSK**

Modulateurs FSK (phase continue) :

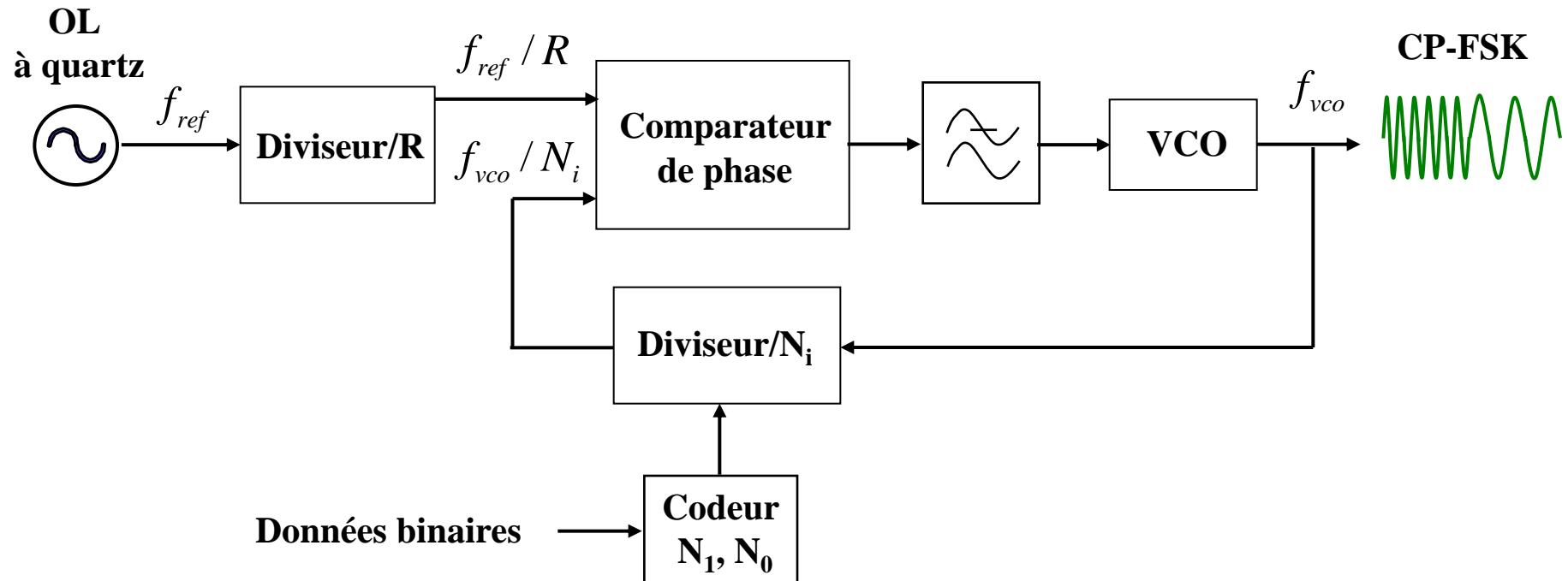


f_1 et f_0 sont issues d'un même VCO piloté par le signal NRZ des données.

Ceci garantit la **continuité de phase** aux sauts en fréquence.



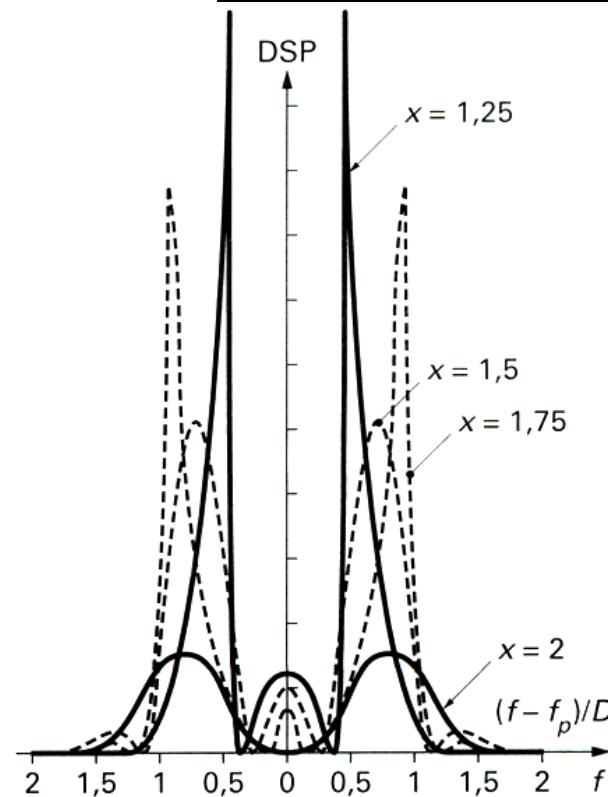
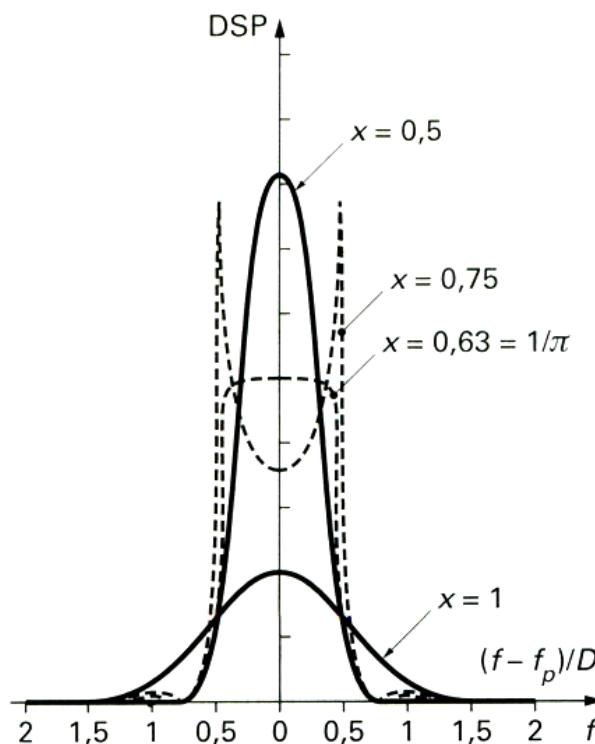
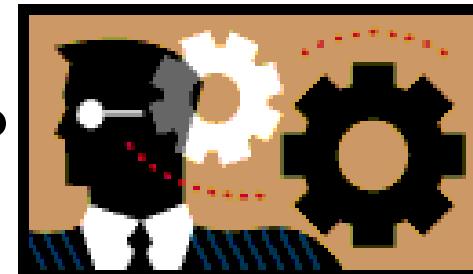
Modulateur FSK à PLL (phase continue) :



$$\text{PLL verrouillée} \Rightarrow \frac{f_{ref}}{R} = \frac{f_{vco}}{N_i} \Rightarrow f_{vco} = \frac{N_i}{R} f_{ref}$$

- DSP d'un signal CP-FSK

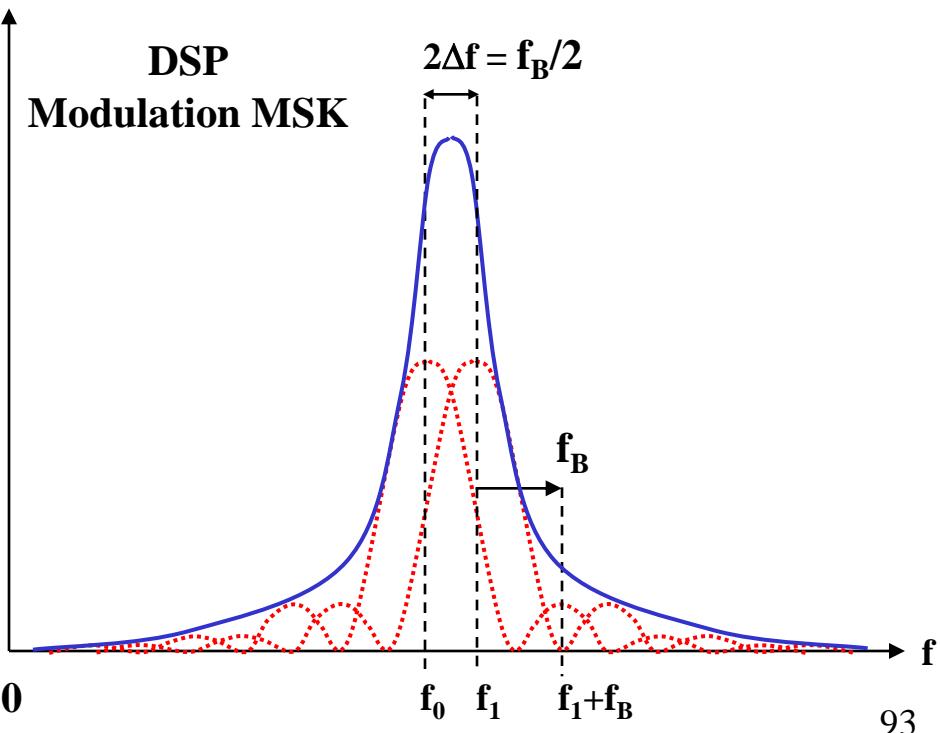
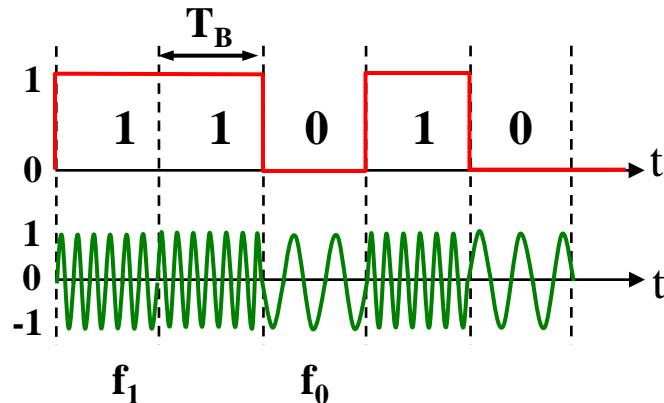
Le calcul est, comment dire ? 



- MSK (Minimum Shift Keying)

La modulation MSK est le cas particulier de FSK où l'intervalle entre f_1 et f_0 est minimum (en deçà la détection n'est plus possible).

$$(f_1 - f_0)/f_B = 0,5 \quad \text{où } f_B = 1/T_B$$



Une MSK peut être vue comme une modulation de phase.

Codage d'un 0 $\text{Acos}(2\pi f_0 t) = \text{Acos}[2\pi(f - \Delta f)t]$
 $= \text{Acos}[2\pi(f - f_B/4)t] = \text{Acos}[2\pi ft - \Phi_0]$ où $\Phi_0 = (\pi f_B/2)t$ avec $t \in [0 T_B]$

Codage d'un 1 $\text{Acos}(2\pi f_1 t) = \text{Acos}[2\pi(f + \Delta f)t]$
 $= \text{Acos}[2\pi(f + f_B/4)t] = \text{Acos}[2\pi ft - \Phi_1]$ où $\Phi_1 = -(\pi f_B/2)t$ avec $t \in [0 T_B]$

• Modulateur MSK

La MSK pouvant être vue comme une modulation de phase, comme toute modulation de ce type, elle peut être réalisée au moyen d'un **modulateur IQ**.

$$s(t) = \cos(\omega_0 t - \Phi_k) \quad \text{Signal modulé en phase}$$

Or $s(t) = \cos(\Phi_k) \cos(\omega_0 t) + \sin(\Phi_k) \sin(\omega_0 t)$

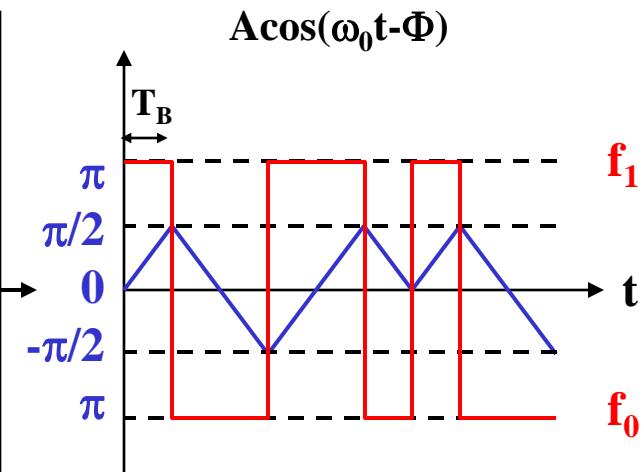
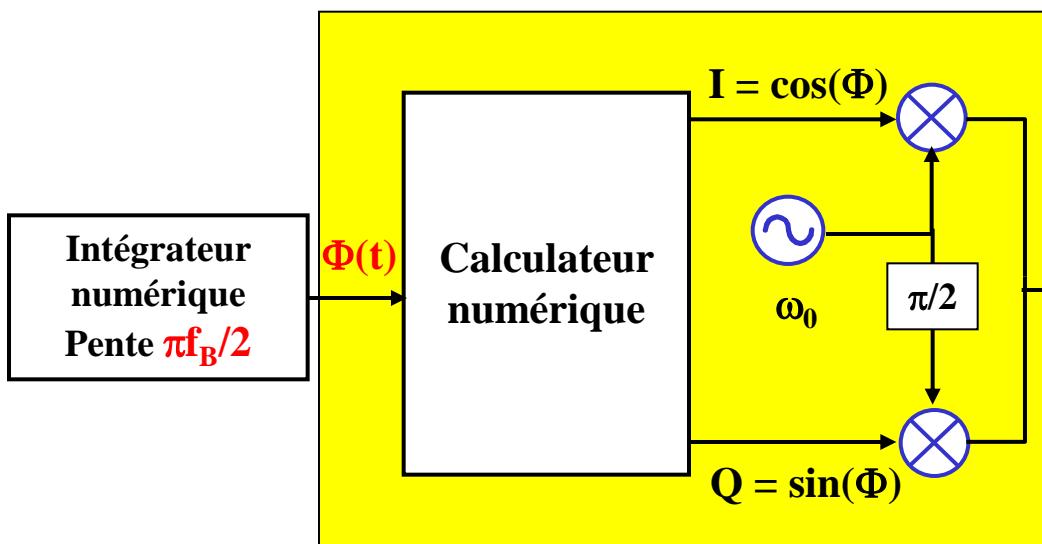
$$s(t) = \underline{I} \cos(\omega_0 t) + \underline{Q} \sin(\omega_0 t)$$

In phase In Quadrature

Codage d'un 0 $A\cos(2\pi f_0 t) = A\cos[2\pi(f - \Delta f)t]$
 $= A\cos[2\pi(f - f_B/4)t] = A\cos[2\pi ft - \Phi_0]$ où $\Phi_0 = (\pi f_B/2)t$ avec $t \in [0 T_B]$

Codage d'un 1 $A\cos(2\pi f_1 t) = A\cos[2\pi(f + \Delta f)t]$
 $= A\cos[2\pi(f + f_B/4)t] = A\cos[2\pi ft - \Phi_1]$ où $\Phi_1 = -(\pi f_B/2)t$ avec $t \in [0 T_B]$

Modulateur IQ



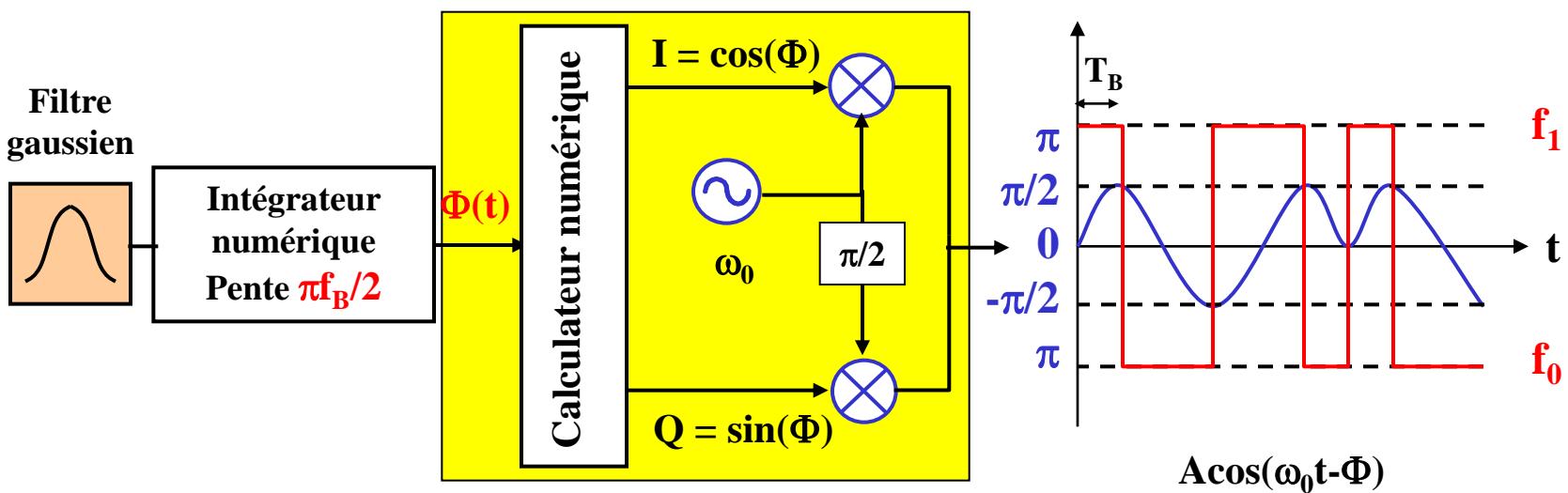
- **GMSK (Gaussian Minimum Shift Keying)**

Pour le **GSM** (téléphone mobile 2G) le format de modulation numérique utilisé est le GMSK (Gaussian Minimum Shift Keying)

Il s'agit d'une MSK précédée d'un filtrage gaussien des données destiné à diminuer la bande passante du signal modulé en atténuant les lobes secondaires du spectre

Du point de vue de la phase du signal modulé, le filtrage gaussien des données permet d'adoucir les transitions ...

Et nous n'avons rien contre un peu de douceur ❤️❤️❤️ n'est-ce pas ?



Transmission sur fréquence porteuse

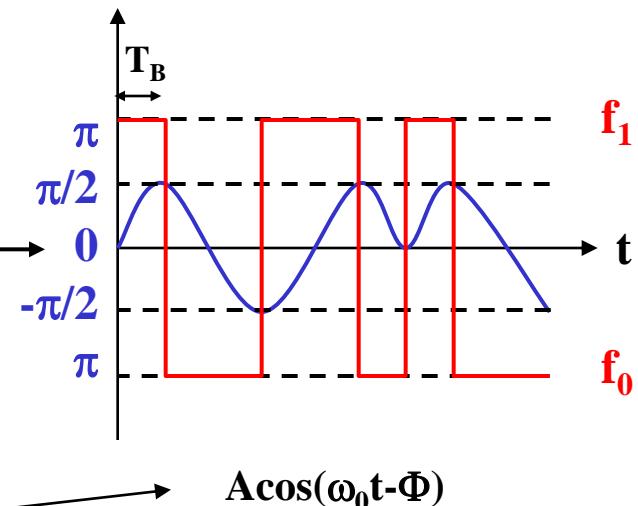
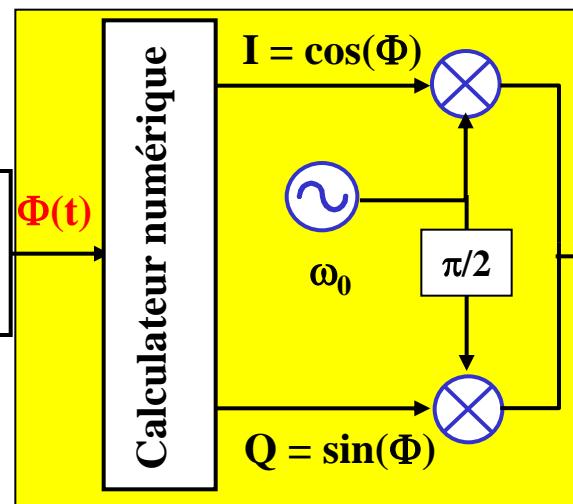
Modulation GMSK

Modulation GMSK

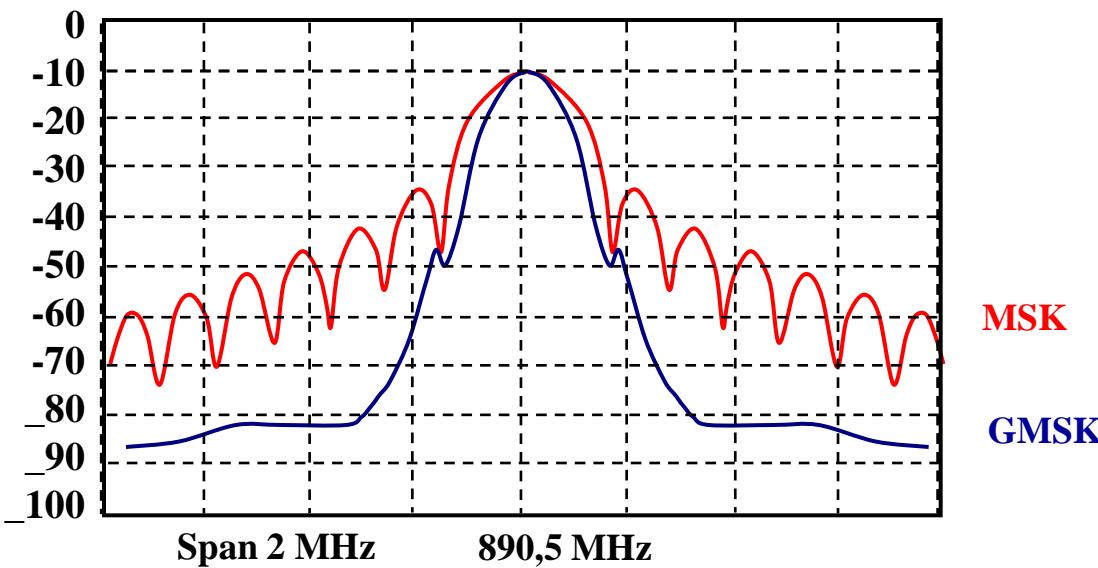
Filtre gaussien



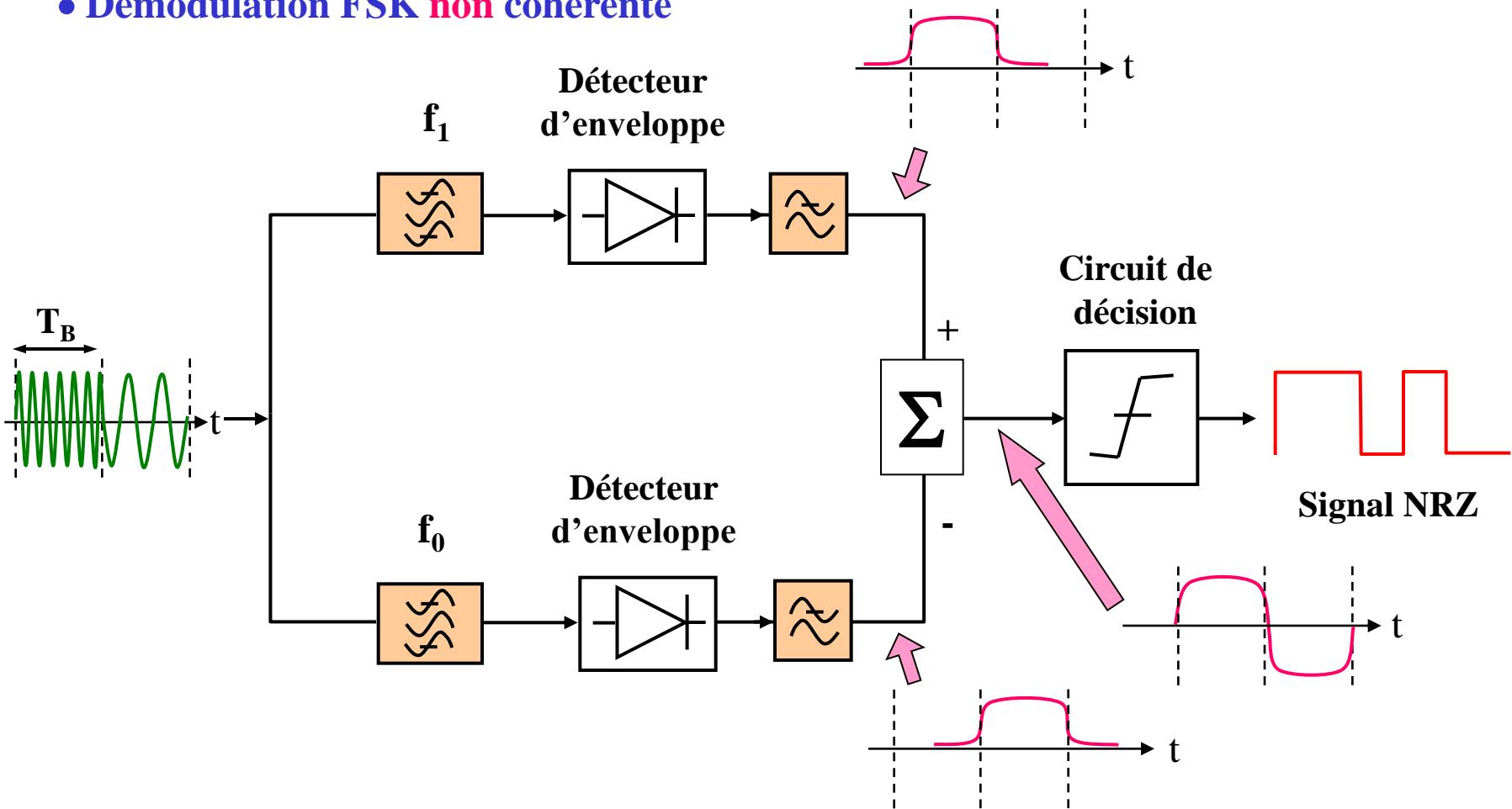
Intégrateur numérique
Pente $\pi f_B / 2$



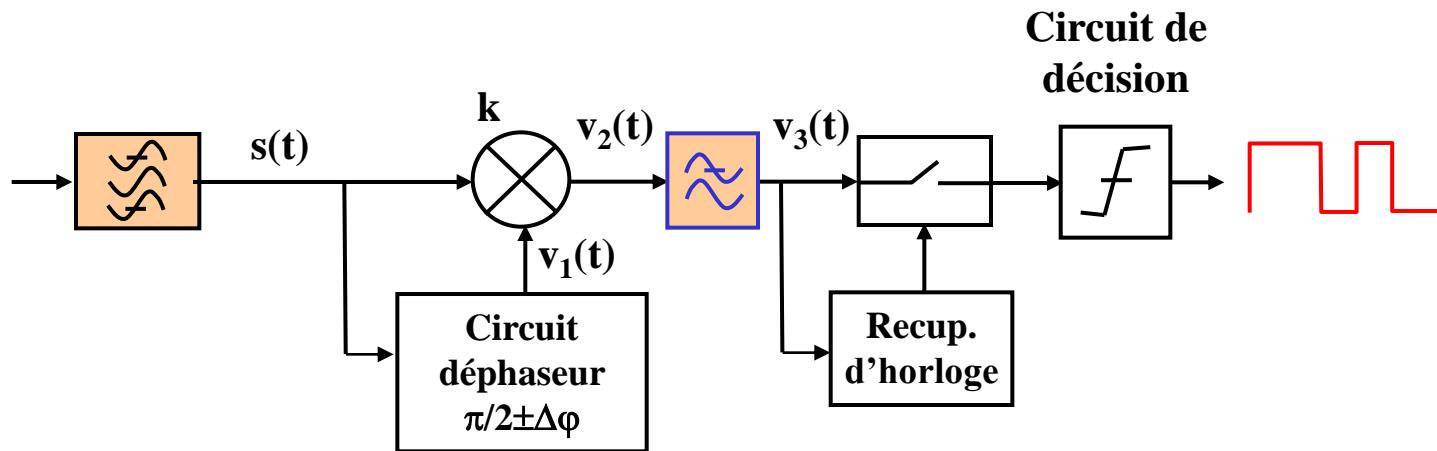
Filtrage gaussien \Rightarrow transitions de phase plus douces diminution de l'occupation spectrale



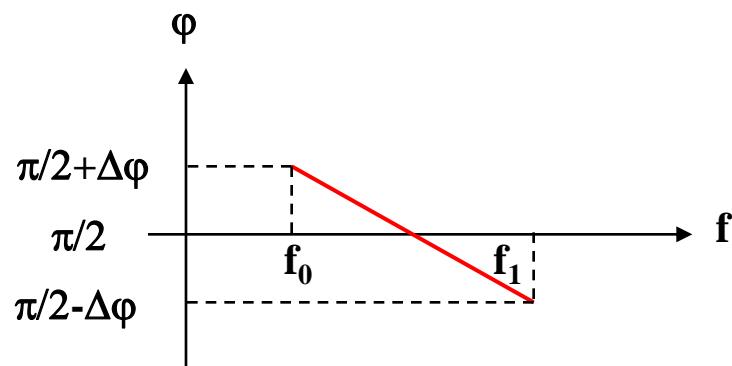
- Démodulation FSK non cohérente



- Démodulateur FSK en quadrature



Caractéristique du déphaseur



Réception d'un 1

$$s(t) = A \cos(2\pi f_1 t) \quad \text{Durant } T_s$$

$$v(t) = A \cos\left(2\pi f_1 t + \frac{\pi}{2} - \Delta\varphi\right)$$

$$v_2(t) = s(t)^* v(t) = k \frac{A^2}{2} \left[\cos\left(2\pi f_1 t + \frac{\pi}{2} - \Delta\varphi\right) + \cos\left(\frac{\pi}{2} - \Delta\varphi\right) \right]$$

$$v_3(t) = -k \frac{A^2}{2} \sin(\Delta\varphi) \quad \text{Durant } T_s$$

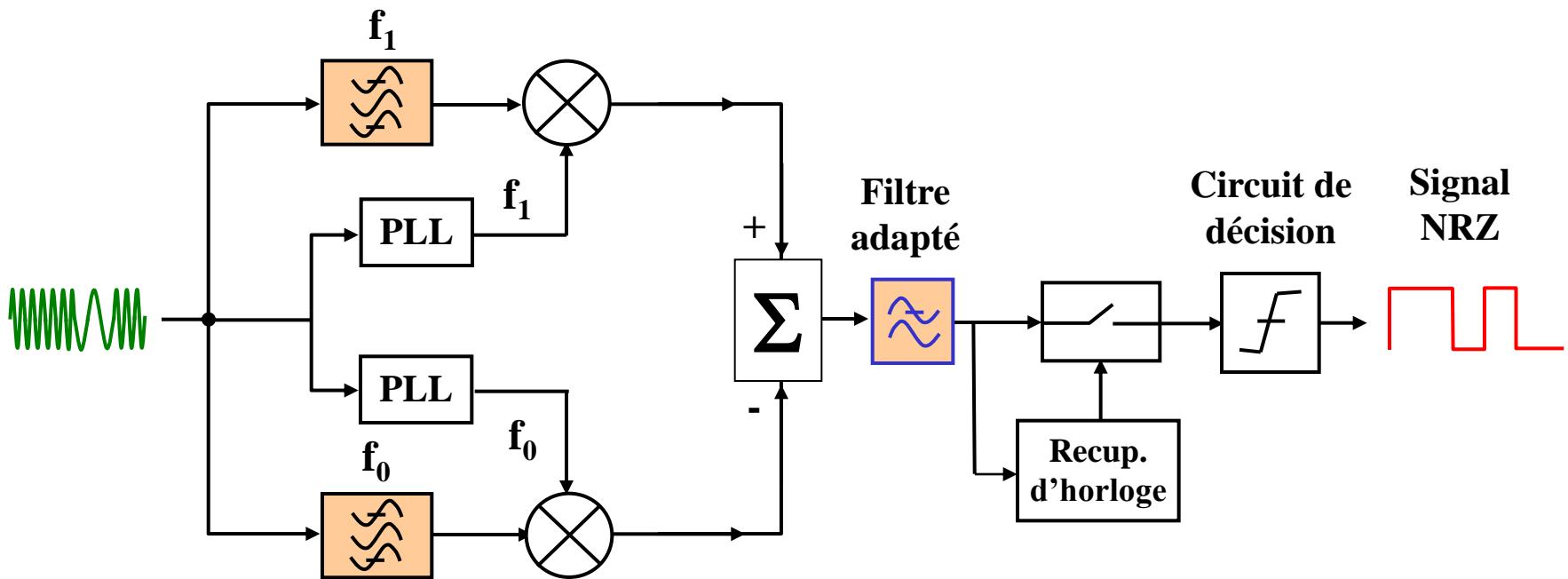
On a deux niveaux...

Réception d'un 0

$$v_3(t) = +k \frac{A^2}{2} \sin(\Delta\varphi)$$

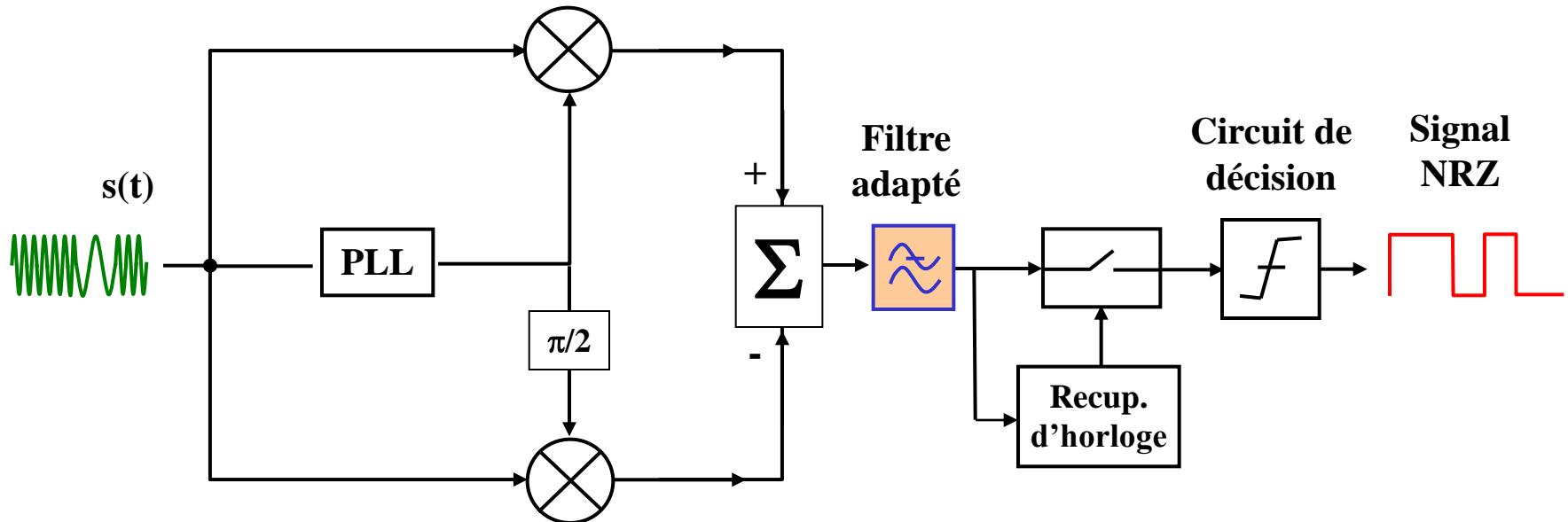
Durant T_s

- Démodulation FSK cohérente



Il faut que les OL soient en phase avec les porteuses f_1 et f_0 émises.

- Démodulation FSK cohérente avec **signaux orthogonaux**



$$f_p = \frac{f_1 + f_0}{2} \quad \Delta f = \frac{|f_1 - f_0|}{2}$$

Ce démodulateur cohérent est conçu pour fonctionner avec des **signaux d'émission $s_1(t)$ et $s_2(t)$ orthogonaux**

The $s_1(t)$ and $s_0(t)$ signals are orthogonal:

$$\int_0^{T_s} s_1(t) s_0(t) dt = 0$$

This implies a condition on f_1 et f_0 :

$$\Rightarrow \int_0^{T_s} A^2 \cos(2\pi f_0 t) \cos(2\pi f_1 t) dt = 0$$

$$\Rightarrow \int_0^{T_s} \frac{A^2}{2} [\cos(2\pi(f_1 + f_0)t) + \cos(2\pi(f_1 - f_0)t)] dt = 0$$

$$\Rightarrow \frac{A^2}{4\pi(f_1 + f_0)} \sin(2\pi(f_1 + f_0)T_s) + \frac{A^2}{4\pi(f_1 - f_0)} \sin(2\pi(f_1 - f_0)T_s) = 0$$

~~$$\Rightarrow \frac{A^2}{4\pi(f_1 + f_0)} \sin(4\pi f_p T_s) + \frac{A^2}{4\pi(f_1 - f_0)} \sin(4\pi \Delta f T_s) = 0$$~~

$$f_p \gg \Delta f$$

$$\Rightarrow \frac{A^2}{4\pi(f_1 - f_0)} \sin(4\pi \Delta f T_s) \square 0$$

Orthogonality
condition

$$\Rightarrow 4\pi\Delta f T_s = n\pi$$

$$\Rightarrow \mu = \frac{2\Delta f}{f_s} = n \times 0.5$$

$$\Rightarrow MSK : \mu_{\min} = 0.5$$

Rque : L'orthogonalité des signaux permet d'optimiser la décision et le TEB

- BER

$$P_{eb} = \frac{1}{2} e^{-\frac{E_b}{2N_0}}$$

Démodulation FSK non cohérente

$$P_{eb} = \frac{1}{2} erfc\left(\sqrt{\frac{E_b}{2N_0}}\right)$$

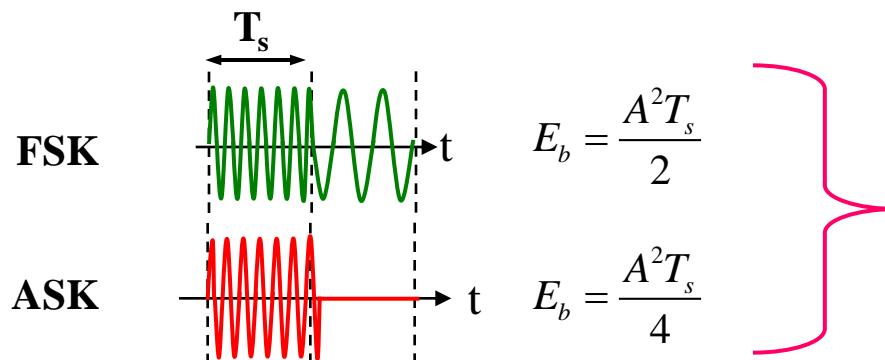
Démodulation FSK cohérente

$$P_{eb} = \frac{1}{2} erfc\left(\sqrt{\frac{E_b}{N_0}}\right)$$

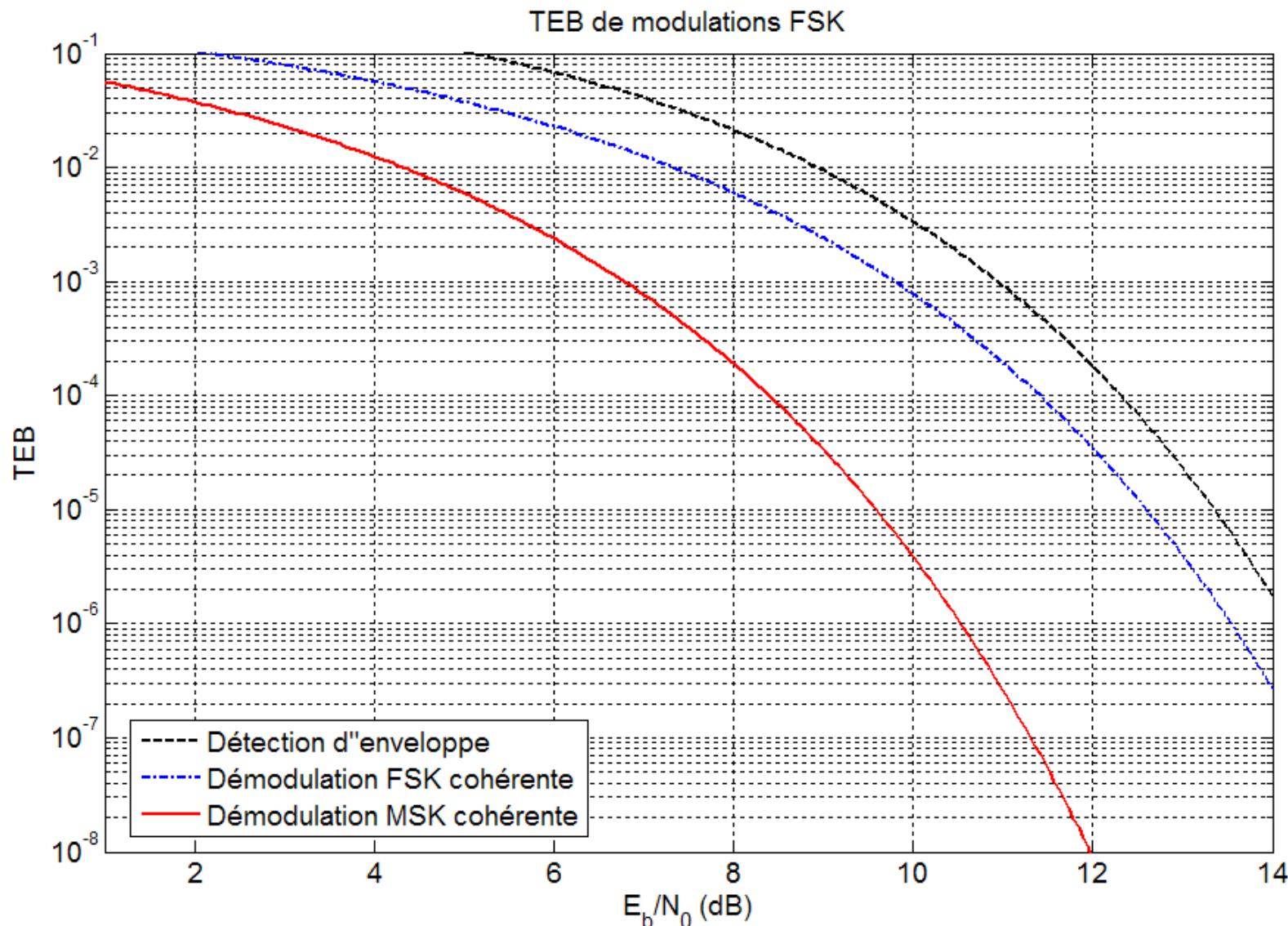
Démodulation MSK cohérente avec signaux orthogonaux

Où E_b est l'**ε moyenne** par bit $E_b = \frac{A^2 T_s}{2}$ A= amplitude des sinusoides f_1 et f_0

Rque : Pour l'ASK on avait une E_b deux fois moindre puisqu'un zéro le porte pas d' ϵ .



Pour comparer deux modulations num.
on suppose une même E_b (moy)...



PSK modulation :

PSK : Phase Shift Keying

For representing a bit or an ensemble of bits one codes the phase Φ_k of the carrier.
Where $k \in [1, M]$ where $M = 2^n$ is the number of possible phase states.

An n-PSK modulation enables coding n bits :

2-PSK ($2 = 2^1$) on code 1 bit

4-PSK ($4 = 2^2$) on code 2 bits

8-PSK ($8 = 2^3$) on code 3 bits

2-PSK \Leftrightarrow BPSK : Binary Phase Shift Keying

4-PSK \Leftrightarrow QPSK : Quadrature Phase Shift Keying

A PSK can be considered as the superposition of 2 ASK on 2 carriers in quadrature.

$$A\cos(\omega t - \Phi_k) = A\cos(\Phi_k) \cos(\omega_0 t) + A\sin(\Phi_k) \cos(\omega_0 t + \pi/2)$$

$$= a_k A\cos(\omega_0 t) + b_k A\cos(\omega_0 t + \pi/2) \quad \text{avec : } a_k = \cos(\Phi_k) \\ b_k = \sin(\Phi_k)$$

- Modulation MDP-2 (modulation par déplacement de phase à 2 états) ou 2-PSK

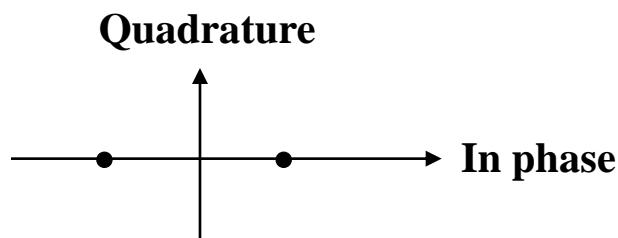
$$s(t) = \sum_k \left[A \cos(\omega_p t + \Phi_k) h(t - kT) \right]$$

En choisissant $0 \rightarrow \Phi_k = 0$; $1 \rightarrow \Phi_k = \pi$

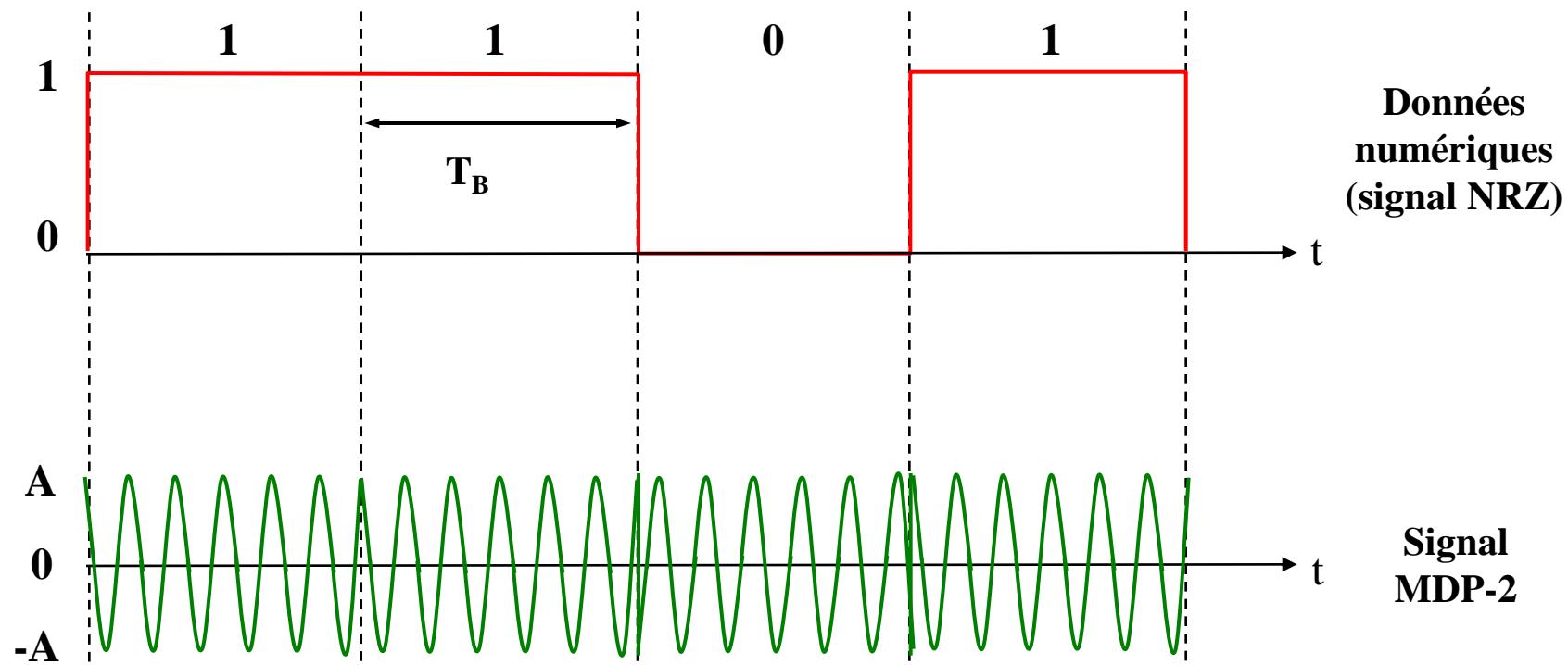
$$\Rightarrow s(t) = \left[\sum_k a_k h(t - kT) \right] A \cos \omega_p t \quad \text{avec} \quad a_k \in \{-1; 1\}$$

MDP-2 : on \times la porteuse par ± 1

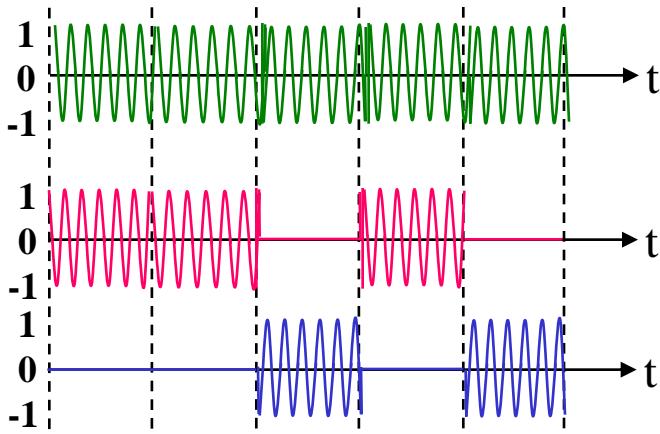
MDP-2 : constellation



MDP-2 : signal temporel



MDP-2 : spectre



On peut considérer le MDP-2 comme la superposition de 2 ASK de deux porteuses en opposition de phase par deux signaux en bande de base de débit $1/T_s$

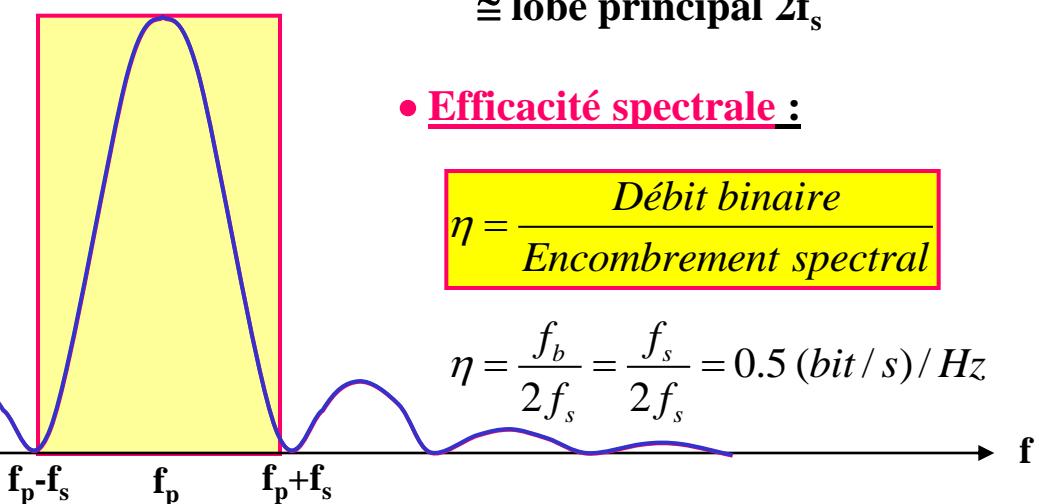
Le spectre de la PSK est la superposition des deux spectres ASK à la même fréquence ...

- Encombrement spectral :
 \cong lobe principal $2f_s$

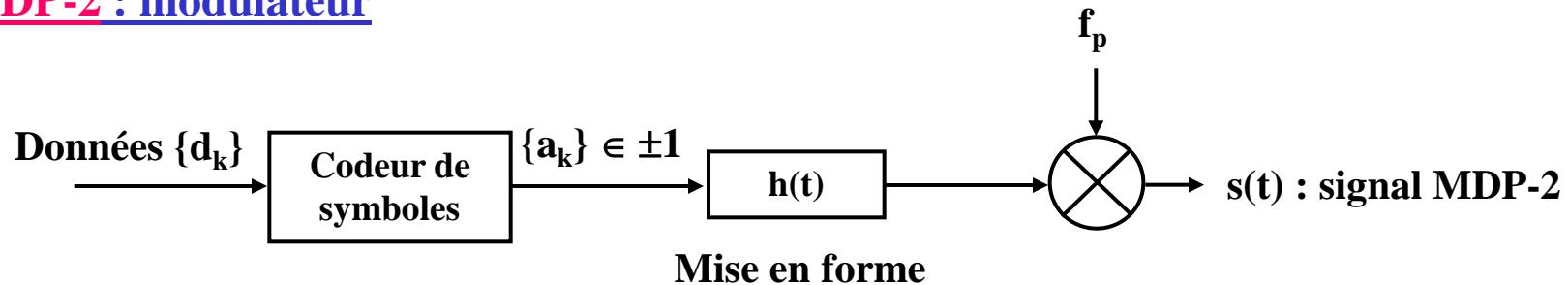
- Efficacité spectrale :

$$\eta = \frac{\text{Débit binaire}}{\text{Encombrement spectral}}$$

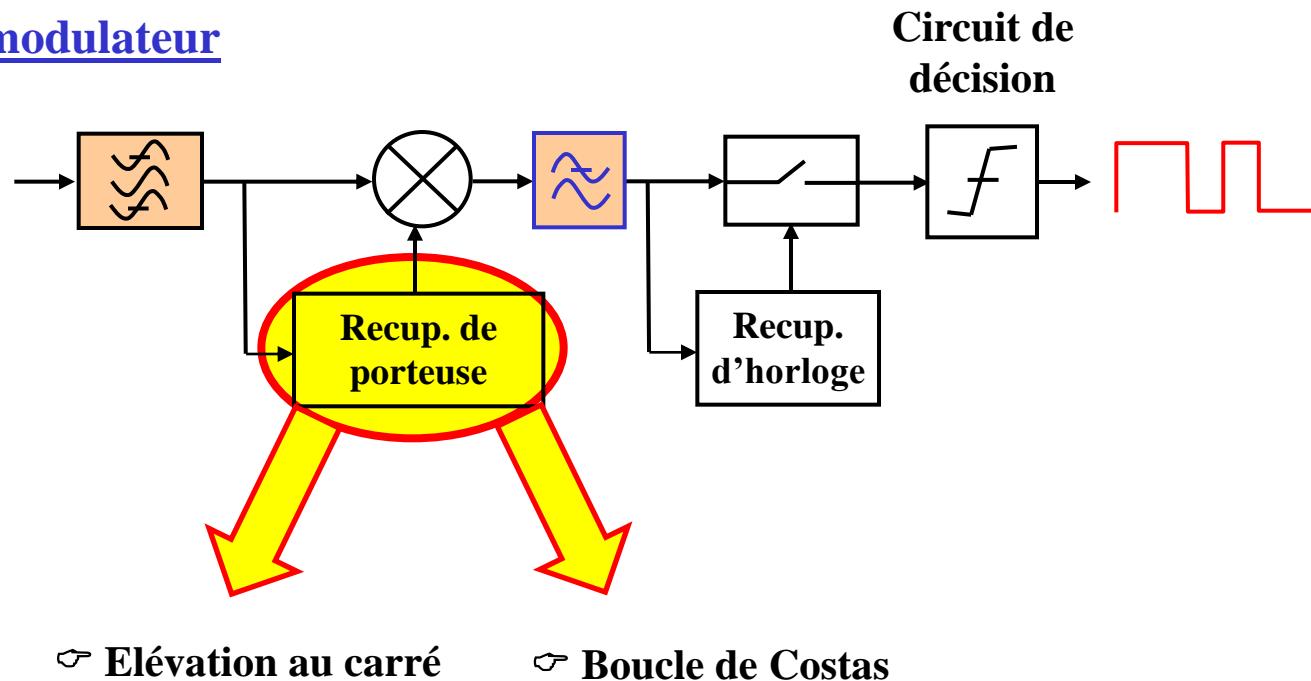
$$\eta = \frac{f_b}{2f_s} = \frac{f_s}{2f_s} = 0.5 \text{ (bit / s) / Hz}$$



MDP-2 : modulateur



MDP-2 : démodulateur

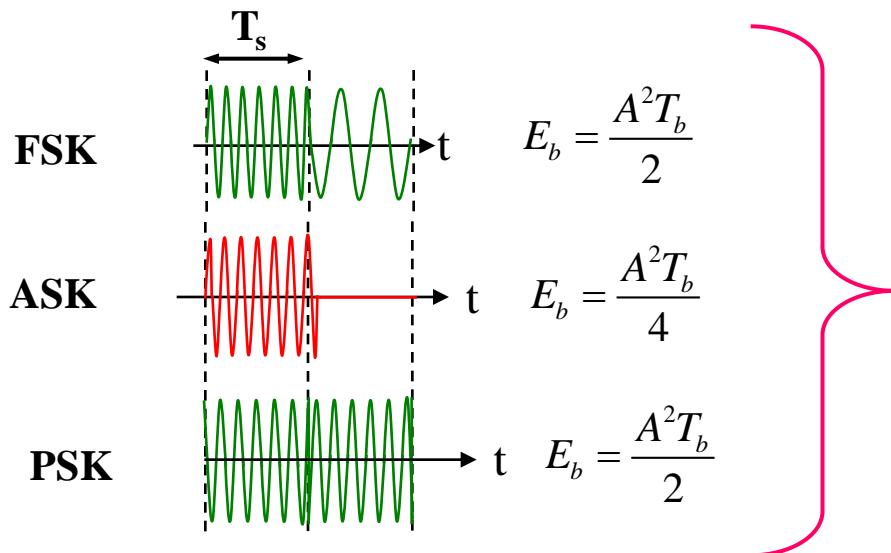


MDP-2 : BER

$$P_{eb} = \frac{1}{2} erfc \left(\sqrt{\frac{E_b}{N_0}} \right)$$

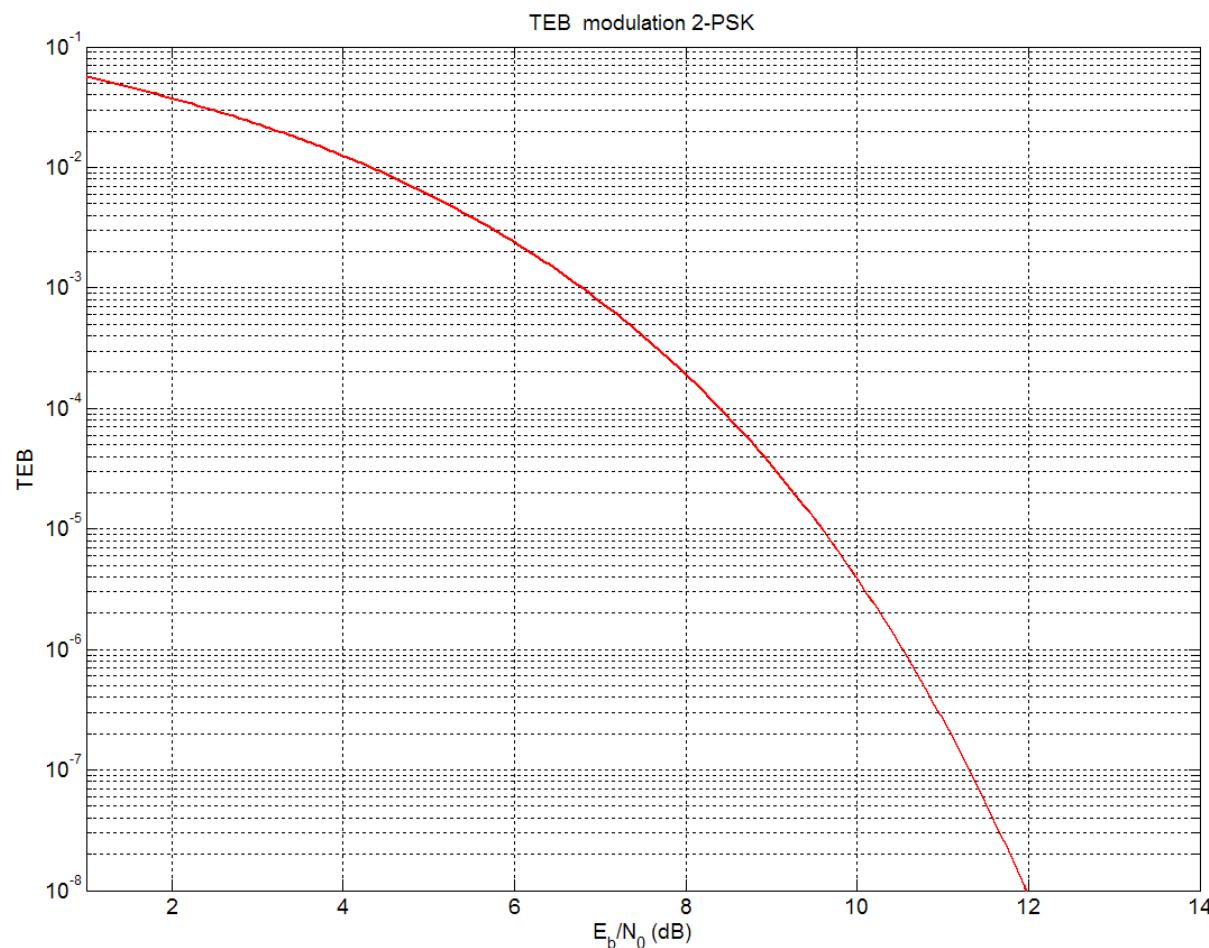
Où E_b est l'**e moyenne** par bit $E_b = \frac{A^2 T_b}{2}$ A= amplitude des sinusoïdes f_p

Rque : Pour l'ASK on avait une E_b deux fois moindre puisqu'un zéro le porte pas d' ϵ .



Pour comparer deux modulations num.
on suppose une même E_b (moy)...

MDP-2 : BER



- Modulation MDP-4 (modulation par déplacement de phase à 4 états) ou QPSK

$$s(t) = \sum_k \left[A \cos(\omega_p t + \Phi_k) h(t - kT_s) \right] \quad \Phi_k \in \left\{ \frac{\pi}{4}; \frac{3\pi}{4}; \frac{5\pi}{4}; \frac{7\pi}{4} \right\}$$

Soit, en décomposant le signal selon les voies I et Q :

$$s(t) = \left[\sum_k h(t - kT_s) \cos \Phi_k \right] A \cos(\omega_p t) - \left[\sum_k h(t - kT_s) \sin \Phi_k \right] A \sin(\omega_p t)$$

En posant $a_k = \cos \Phi_k$ et $b_k = \sin \Phi_k$

$$s(t) = \left[\sum_k h(t - kT_s) a_k \right] A \cos(\omega_p t) - \left[\sum_k h(t - kT_s) b_k \right] A \sin(\omega_p t)$$

La QPSK est la modulation de deux porteuses en quadrature par deux signaux en bande de base $i(t)$ et $q(t)$.

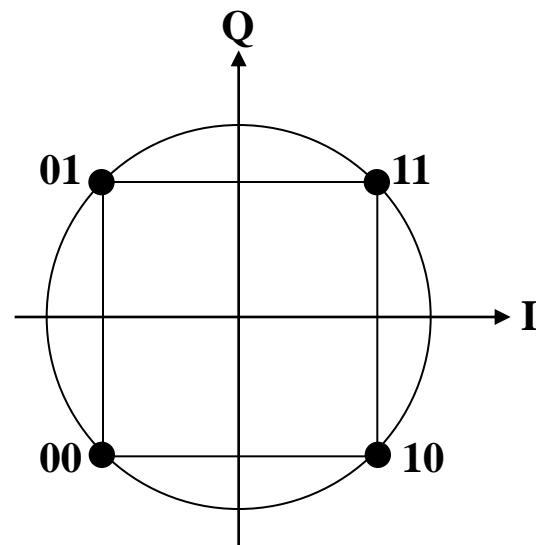
$$i(t) = \sum_k h(t - kT_s) a_k$$

$$q(t) = \sum_k h(t - kT_s) b_k$$

	a_k	b_k	Φ
11	$\sqrt{2}/2$	$\sqrt{2}/2$	$\pi/4$
01	$-\sqrt{2}/2$	$\sqrt{2}/2$	$3\pi/4$
00	$-\sqrt{2}/2$	$-\sqrt{2}/2$	$5\pi/4$
10	$\sqrt{2}/2$	$-\sqrt{2}/2$	$7\pi/4$

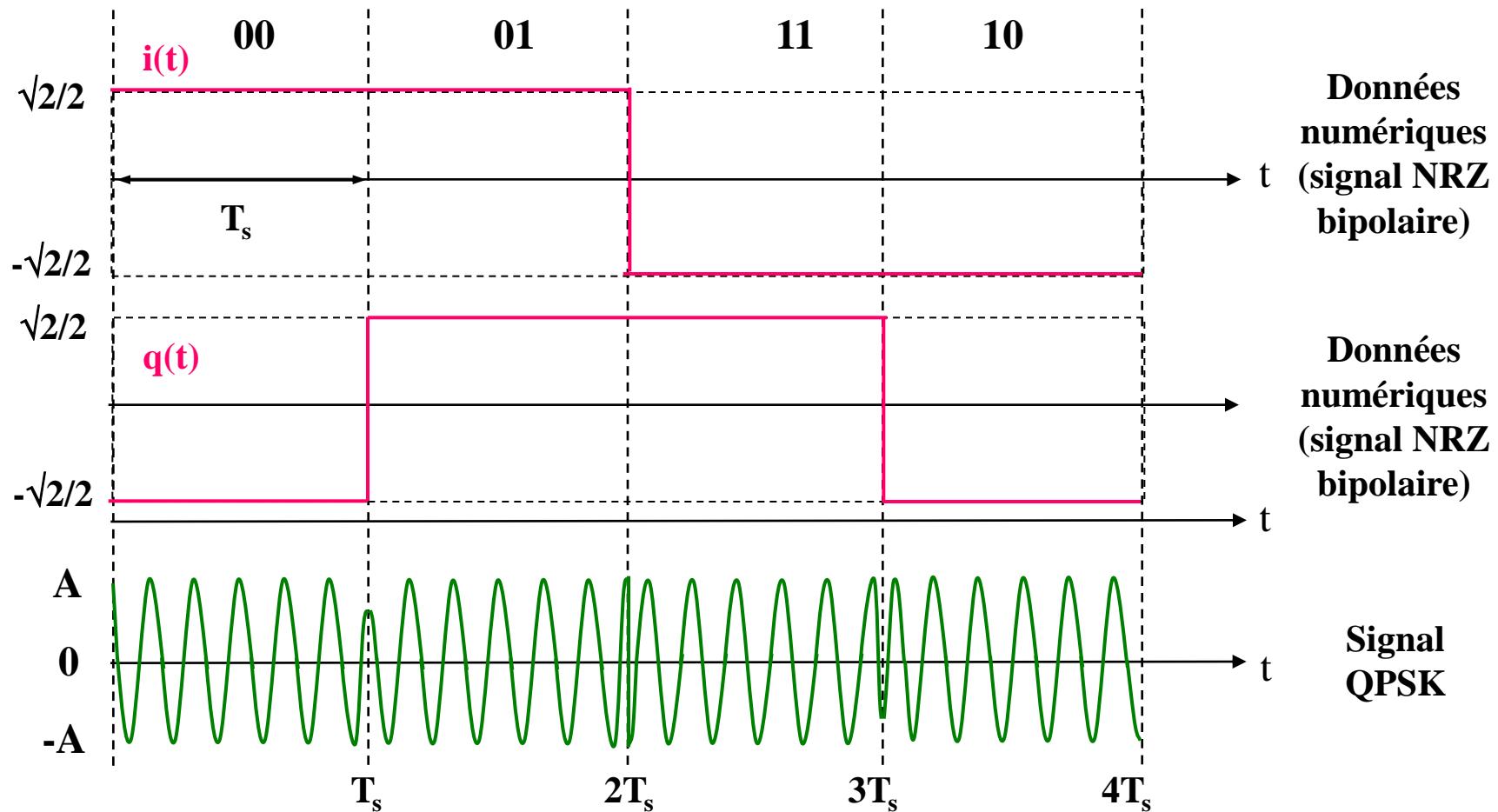
D'après le tableau qui précède $i(t)$ et $q(t)$ sont deux signaux NRZ bipolaires...

QPSK : constellation



D'après la constellation, les sauts de phase d'un symbole à l'autre peuvent être de $\pm\pi/2$ et π .

QPSK : signal temporel

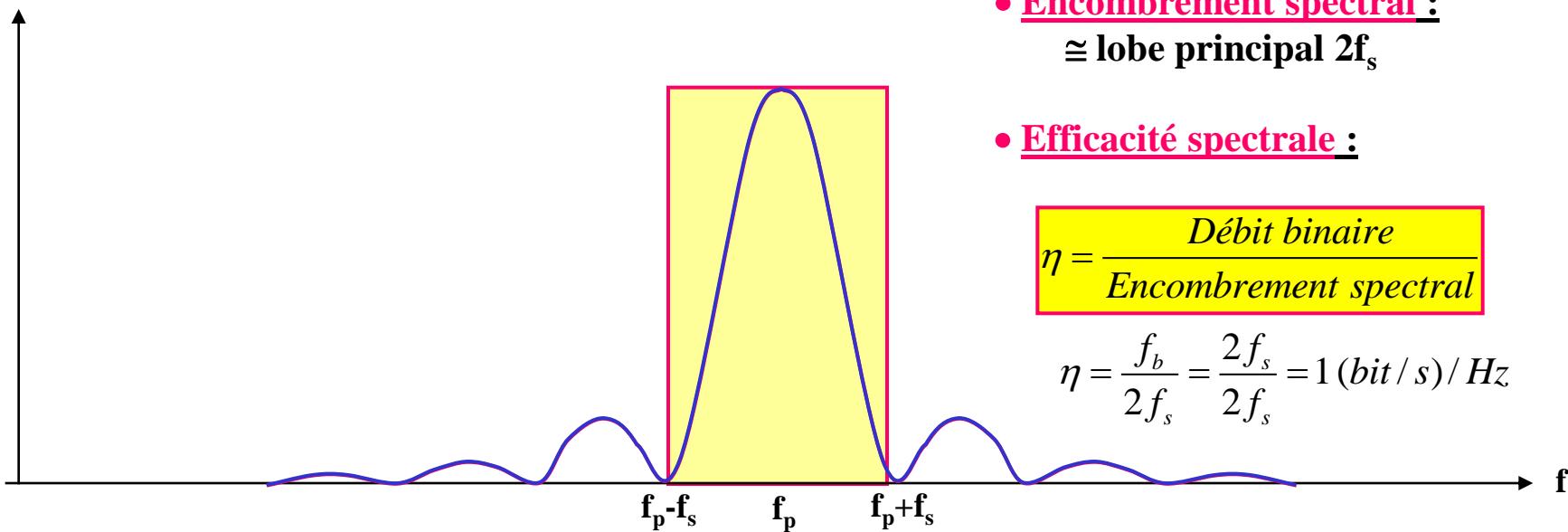


Les sauts de phase d'un symbole à l'autre peuvent être de $\pm\pi/2$ et π

QPSK : spectre

On peut considérer la QPSK comme la superposition de 2 ASK de deux porteuses en quadrature par deux signaux en bande de base de débit $1/T_s$

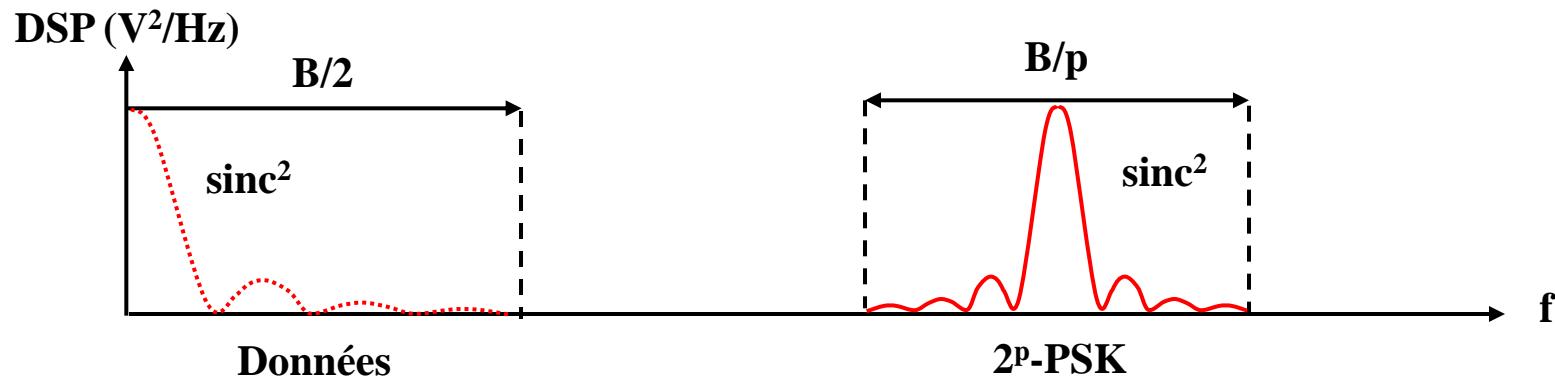
Les données sur les voies I et Q étant décorrélées le spectre de la PSK est la superposition des deux spectres ASK à la même fréquence ...



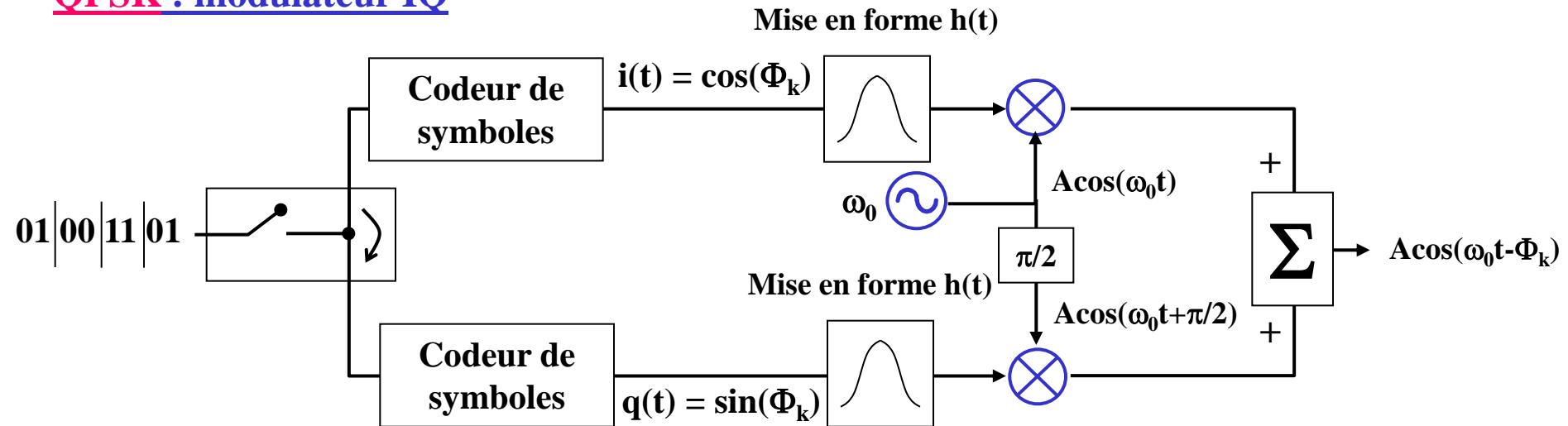
Remarque : Spectre d'une M-PSK

D'une manière générale le spectre d'une M-PSK ne dépend pas du nombre n de bits transmis par un symbole.

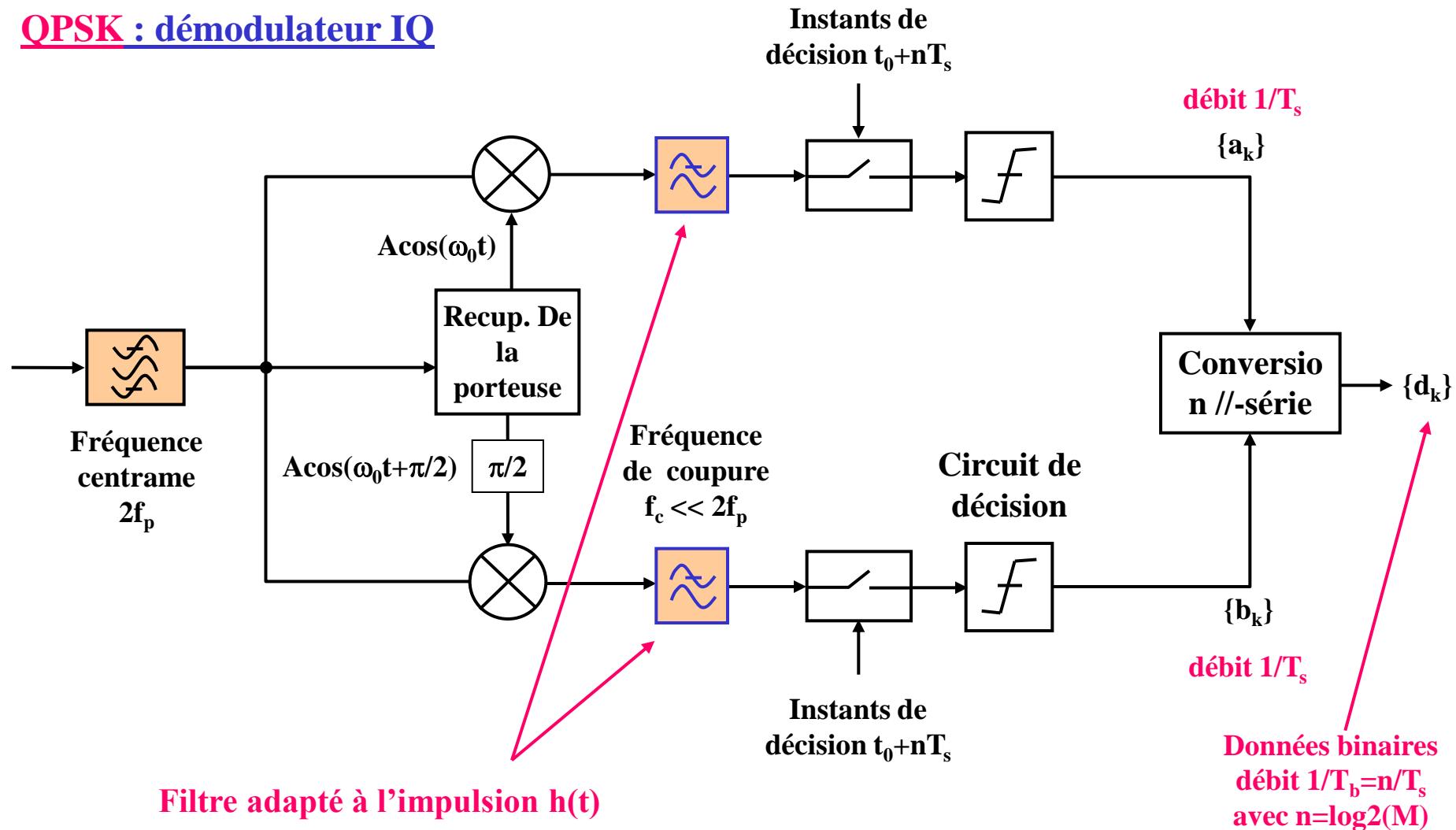
Si on se place à T_s fixé, l'efficacité spectrale (bit/s)/Hz d'une modulation M-PSK est n fois ($m = 2^n$) celle d'une modulation 2-PSK.



QPSK : modulateur IQ



QPSK : démodulateur IQ

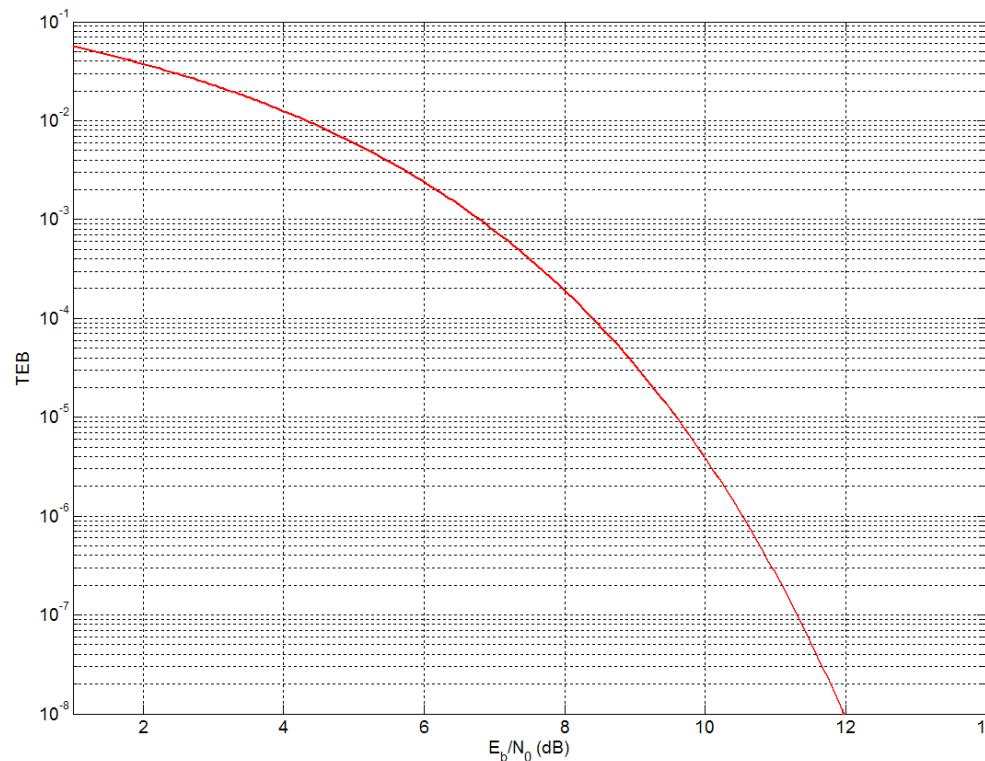


QPSK : BER

$$P_{eb} = \frac{1}{2} erfc\left(\sqrt{\frac{E_b}{N_0}}\right)$$

comme MDP2...

Où E_b est l'**ε moyenne par bit** $E_b = \frac{A^2 T_b}{2}$ A= amplitude des sinusoïdes f_p

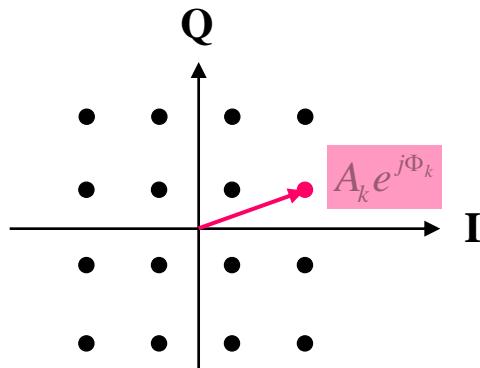


Modulation MAQ-M : *Modulation d'amplitude en quadrature à M états.*

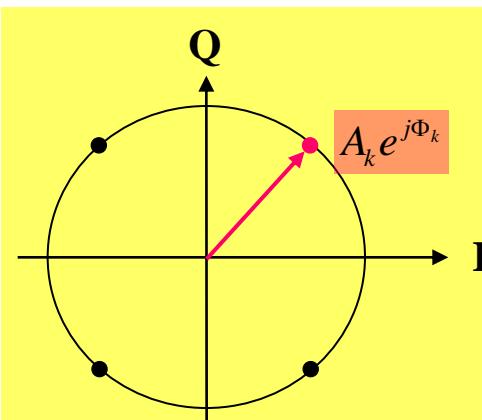
Signal modulé

$$s(t) = A \sum_k [a_k \cos 2\pi f_0 t + b_k \sin 2\pi f_0 t] h(t)$$

Point de la constellation $A_k e^{j\Phi_k}$ avec $A_k = \sqrt{a_k^2 + b_k^2}$ et $\Phi_k = \text{atan}\left(\frac{b_k}{a_k}\right)$



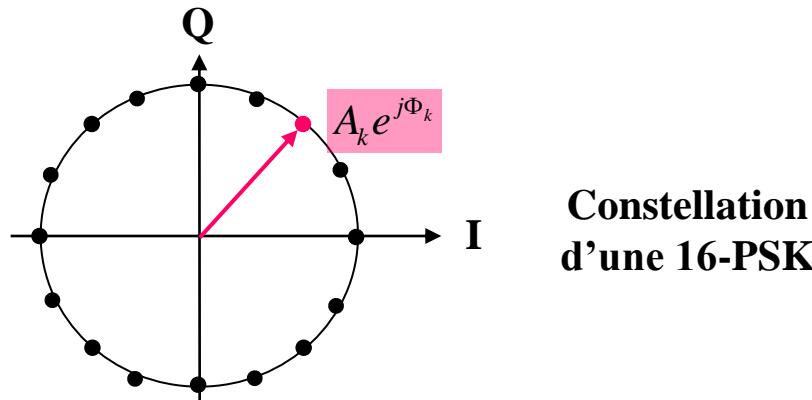
Constellation d'une MAQ 16



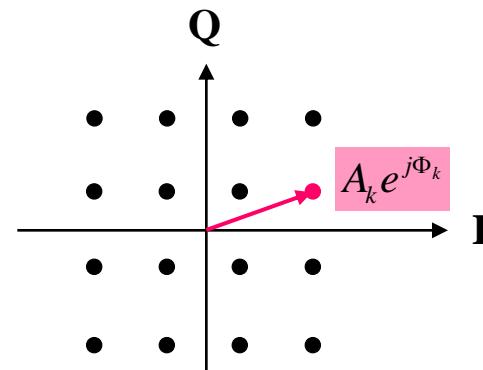
Constellation d'une MDP-4

Remarque : les modulations PSK vues précédemment, peuvent être considérées, et réalisées, comme la modulation de deux porteuses I (en phase) et Q (en quadrature) par des symboles a_k et b_k respectivement. Pour une PSK le module $A_k = \sqrt{a_k^2 + b_k^2} = C^{te}$.

Remarque : En augmentant la valence d'une PSK on rapproche les points de la constellation et on augmente le risque d'erreur (sauf à d'augmenter le rayon $A = \sqrt{a_k^2 + b_k^2}$ cad l' ϵ moyenne par bit E_b).



Une MAQ aux points uniformément répartis dans le plan de Fresnel est préférable.



Constellation d'une MAQ 16

$$s(t) = \left[\sum_k h(t - kT_s) a_k \right] A \cos(\omega_p t) - \left[\sum_k h(t - kT_s) b_k \right] A \sin(\omega_p t)$$

$$i(t) = \sum_k h(t - kT_s) a_k$$

$$q(t) = \sum_k h(t - kT_s) b_k$$

Signaux en bande de base de DSP (cf. cours plus haut...)

$$\gamma_i(f) = \gamma_q(f) = \frac{\sigma^2}{T_s} |H(f)|^2$$

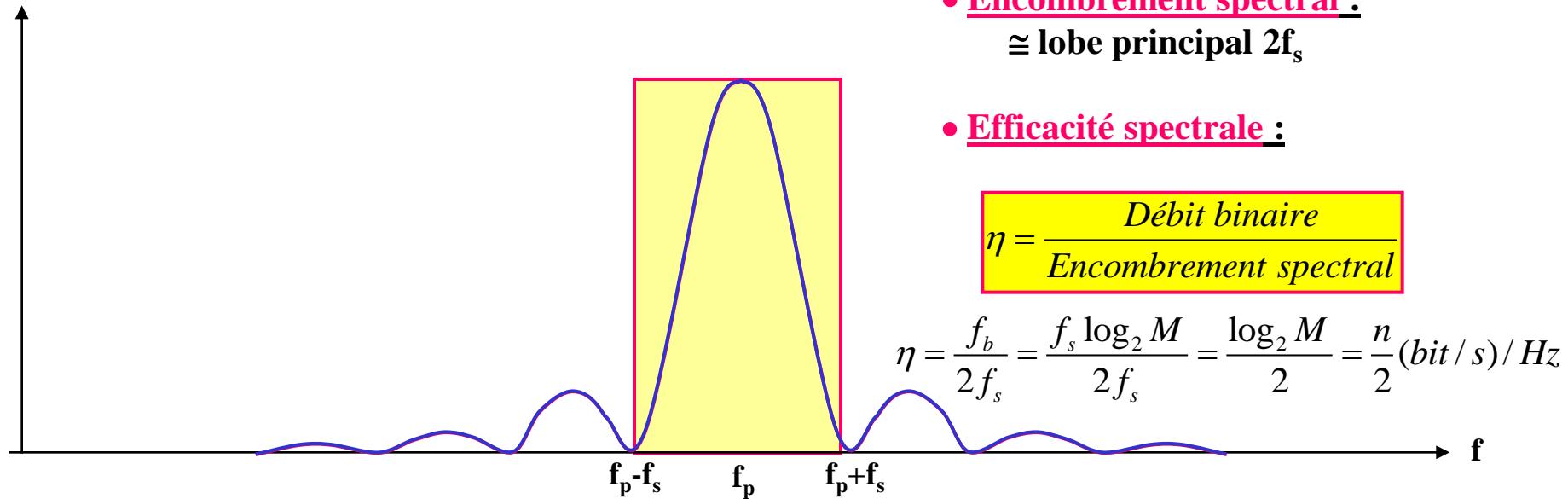
Avec $\rightarrow H(f)$ TF de la forme d'onde $h(t)$

$$\rightarrow \sigma = E[a_k] = E[b_k]$$

Les données sur les voies I et Q étant décorrélées, le spectre d'une MAQ est obtenu par la superposition des transposées autour de f_p des spectres des voies I et Q ...

$$\gamma_s(f) = \gamma_i(f) = \gamma_q(f) = \frac{A^2 T_s^2 \sigma^2}{2 T_s} \left[|H(f - f_p)|^2 + |H(f + f_p)|^2 \right]$$

MAQ : spectre En supposant des impulsions de type NRZ...



- **Modulation et démodulation MAQ**

Même structure que pour la PSK...

- **BER MAQ-M**

“On” montre que

$$P_{eb} = \frac{2}{\log_2 M} \left(\frac{\sqrt{M} - 1}{\sqrt{M}} \right) erfc \left(\sqrt{\frac{3 \log_2 M}{2(M-1)} \frac{E_b}{N_0}} \right)$$

Plus la valence \uparrow plus le BER \uparrow ; il faut donc \uparrow le niveau de signal pour garantir un certain BER.

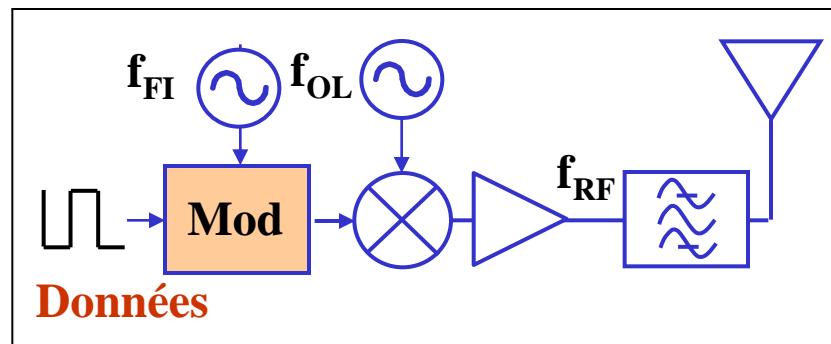
Conclusion

ASK et PSK sont des modulations linéaires qui peuvent être réalisées au moyen de modulateurs IQ...

La FSK est une modulation non linéaire qui présente le désavantage d'une large occupation spectrale (2 porteuses).

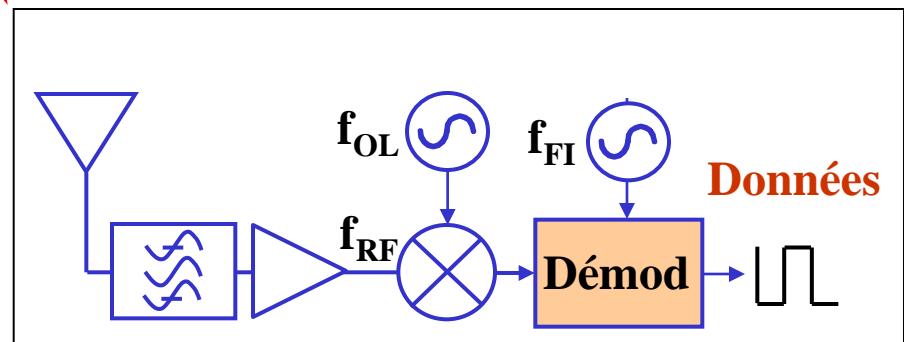
Désormais ce sont le MAQ qui sont utilisées par exemple l'ADSL : MAQ-1024 ($=2^{10}$, codage sur 10 bits) avec un débit de 16-20 Mbit/s ; le VDSL MAQ-32768 ($=2^{15}$, codage sur 15 bits) avec un débit de 56 Mbit/s ...

Emetteur



L'architecture d'un récepteur RF est symétrique de celle de l'émetteur.

Récepteur



Exemple bilan d'une liaison satellite

Rapport signal sur bruit en réception

$$\frac{S}{N} = \frac{P_e G_e A_{el} G_r}{kTB}$$

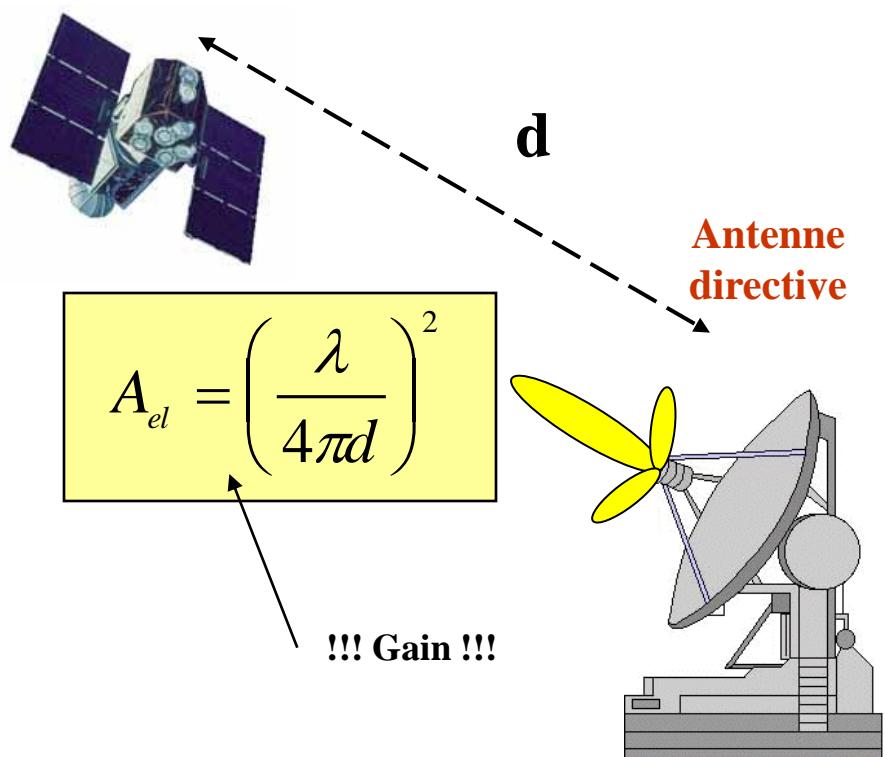
➤ Signal reçu $S = P_e G_e A_{el} G_r$

Pe : puissance émise

Ge : gain de l'antenne d'émission

Ael : atténuation en espace libre

Gr : gain de l'antenne de réception



On pourrait aussi multiplier le signal reçu par l'atténuation atmosphérique à la fréquence utilisée

Exemple bilan d'une liaison satellite

Rapport signal sur bruit en réception

$$\frac{S}{N} = \frac{P_e G_e A_{el} G_r}{kTB}$$

➤ Bruit reçu $N = kTB$

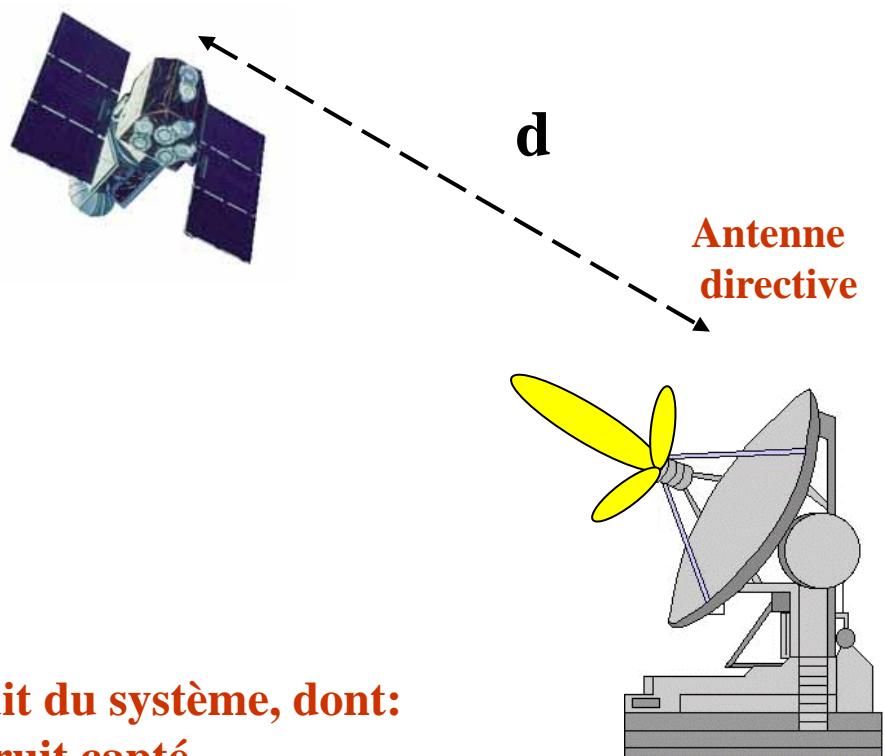
$$T = T_a + T_r$$

B : bande passante du système

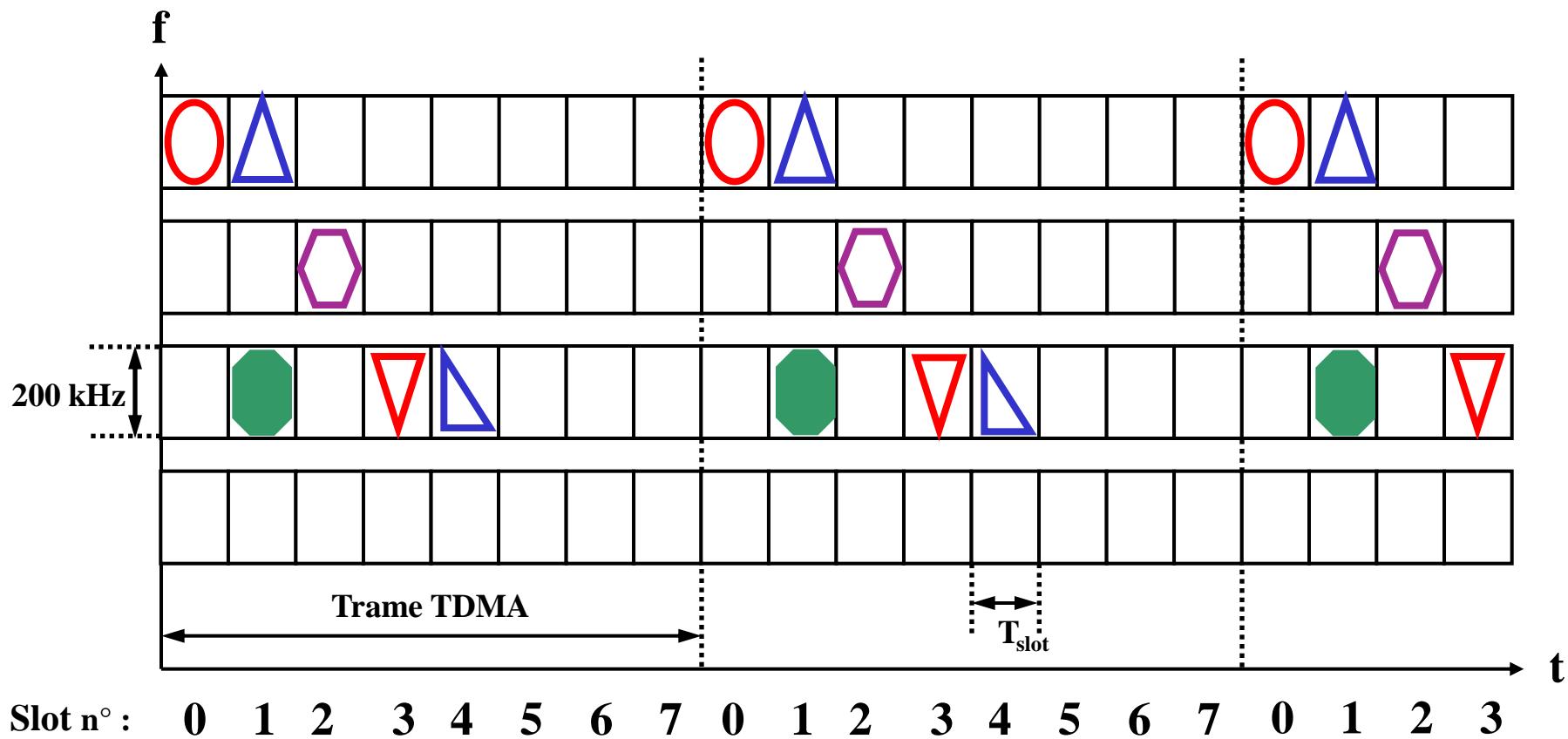
T : température équivalente de bruit du système, dont:

**T_a : Température équivalente du bruit capté
par l'antenne (galactique, environnement)**

**T_r : Température équivalente de bruit produit
par le récepteur**



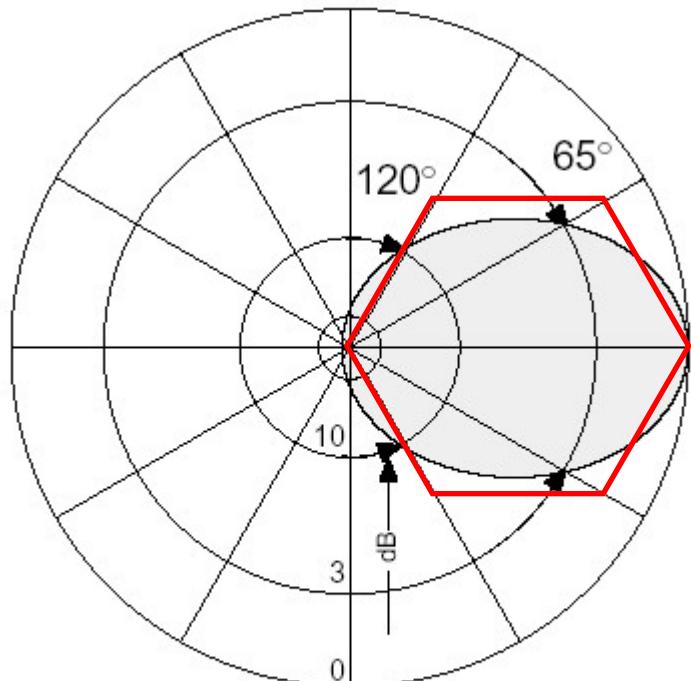
$$T_{\text{slot}} = 0,5769 \text{ ms} \quad T_{\text{TDMA}} = 4,562 \text{ ms} = 8 T_{\text{slot}}$$



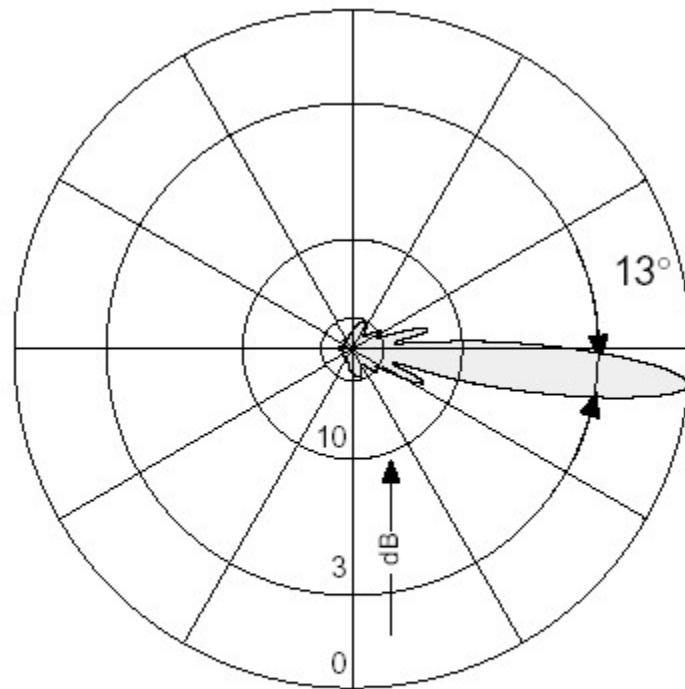
VPol Panel 806–960 65° 15.5dBi 6°T

Type No.	732 691
Frequency range	806 – 960 MHz
Polarization	Vertical
Gain	15.5 dBi
Half-power beam width	H-plane: 65° E-plane: 13°
Electrical downtilt	6°, fixed
Front-to-back ratio	> 25 dB
Impedance	50 Ω
VSWR	< 1.3
Intermodulation IM3 (2 x 43 dBm carrier)	< -150 dBc
Max. power	500 W (at 50 °C ambient temperature)

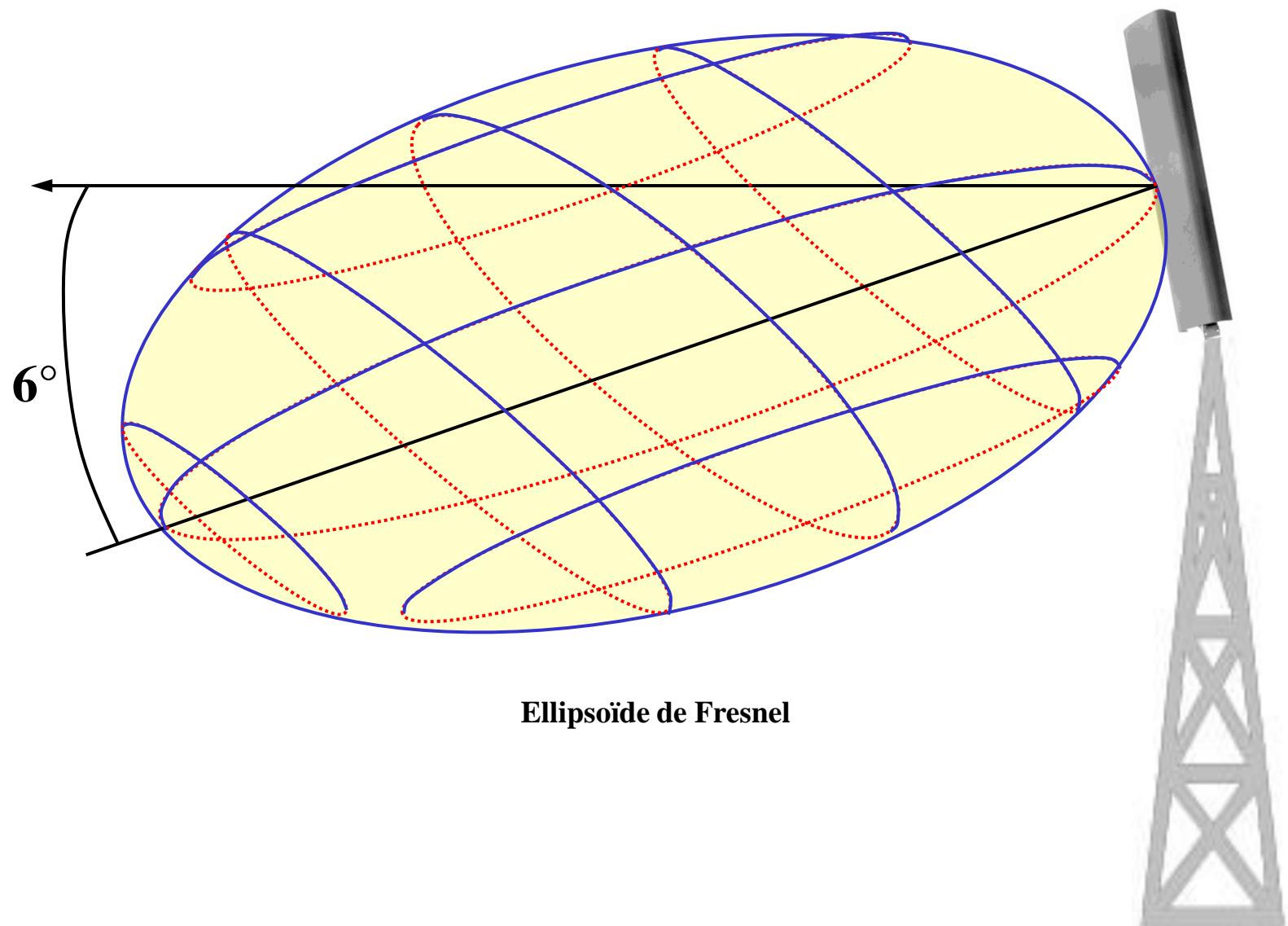




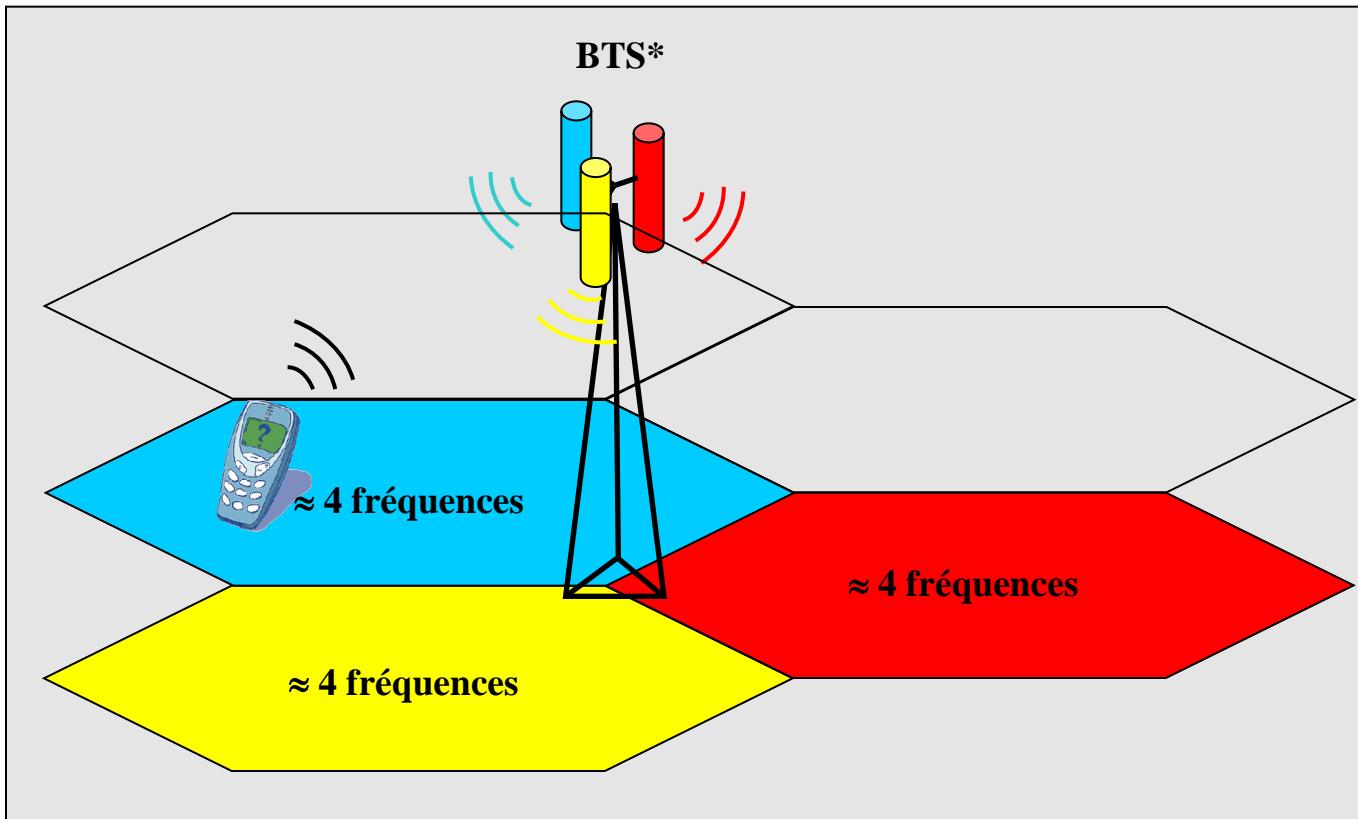
Horizontal Pattern



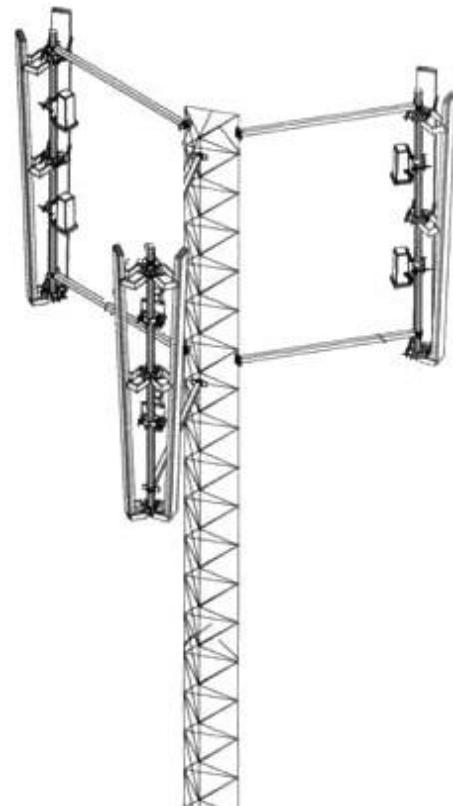
Vertical Pattern
6° electrical downtilt



Ellipsoïde de Fresnel

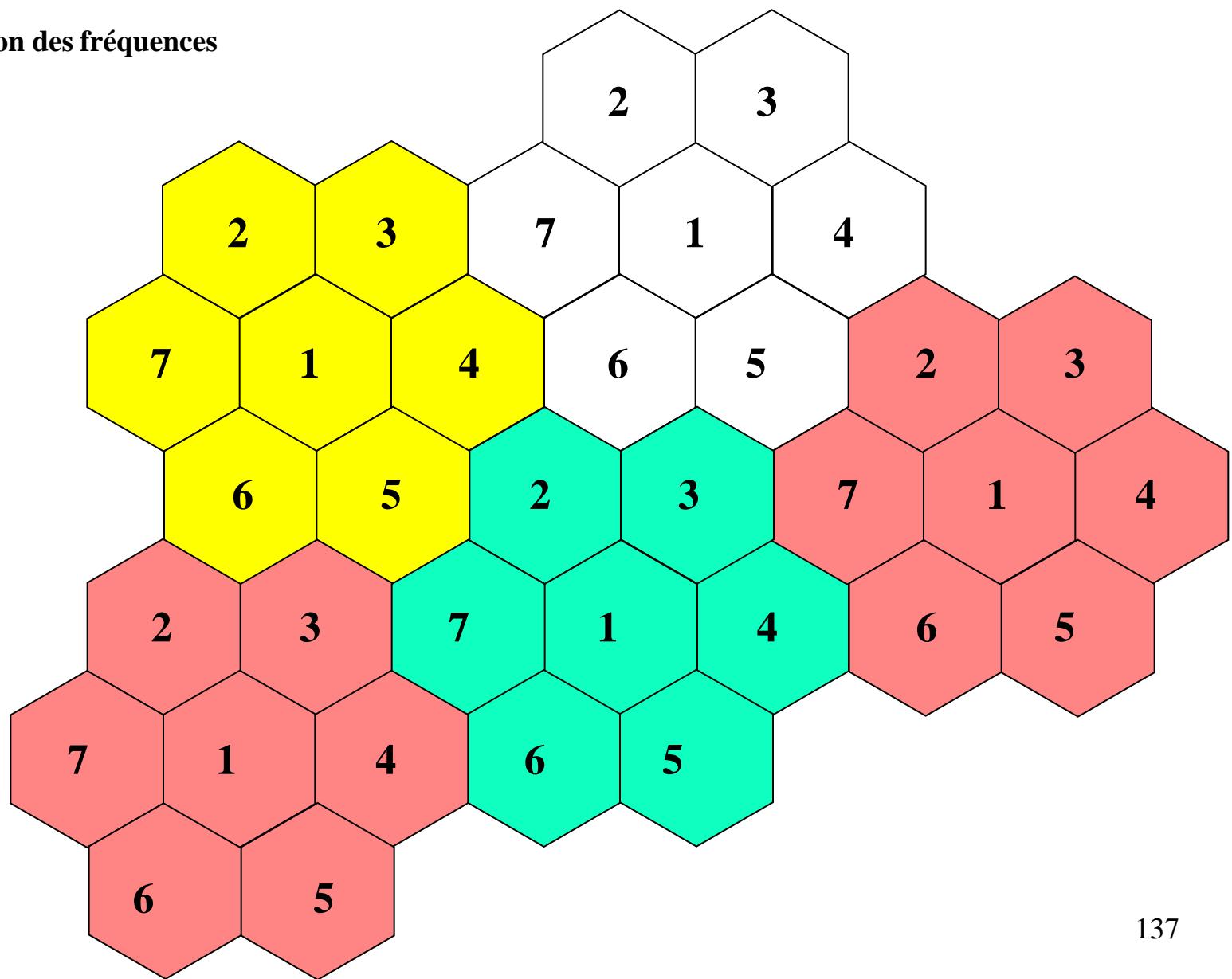


* BTS : Base Station Transceiver (Tx/Rx cad émetteur/récepteur)

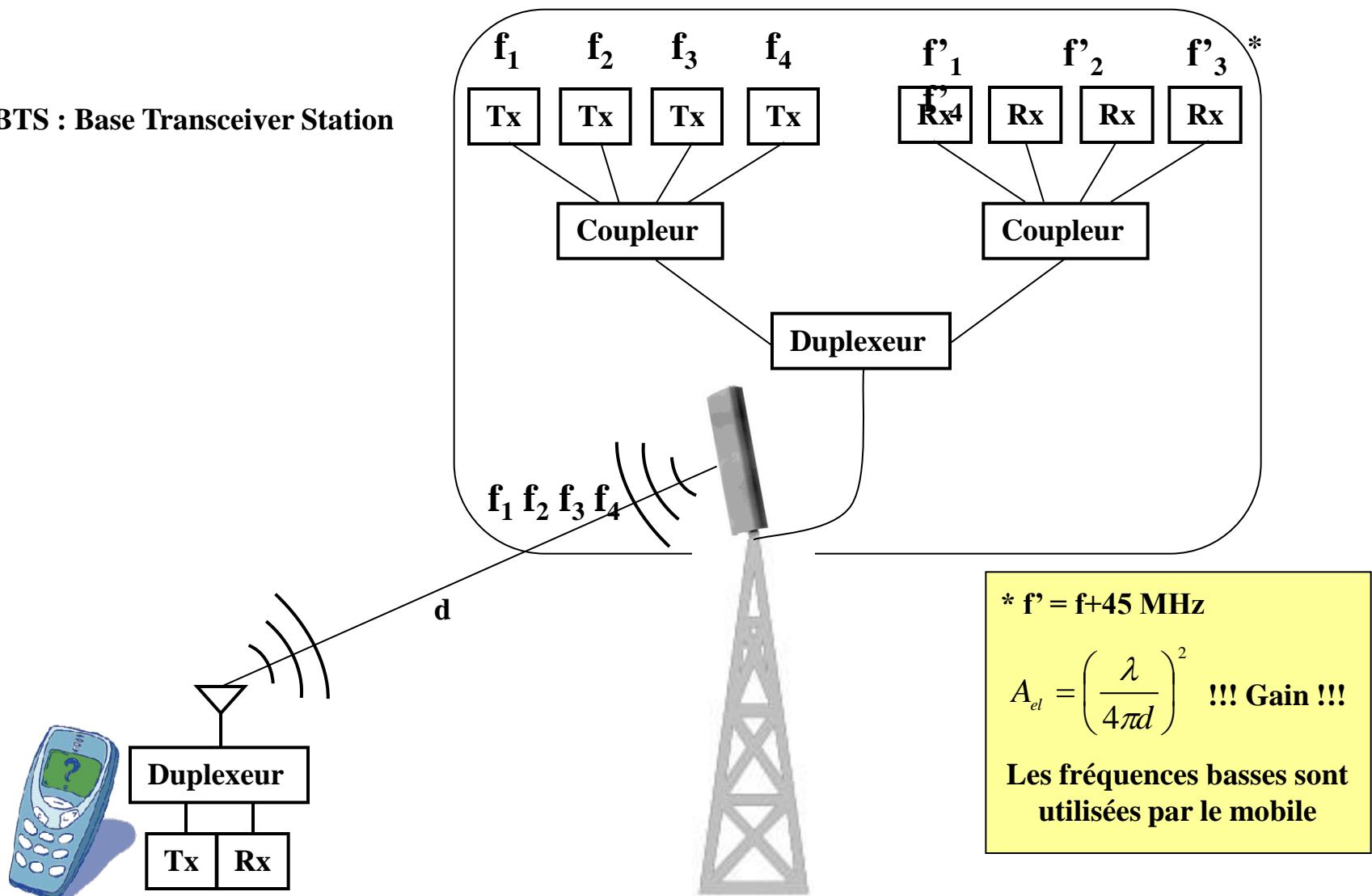


BTS* comportant
 $3 \times 2 = 6$ antennes
(diversité d'antennes)



Réutilisation des fréquences

BTS : Base Transceiver Station



➤ Exemple de système « HF » GSM-DCS

Bilan de liaison

Source : Réseaux GSM-DCS, p 148
 Lagrange, Godlewski, Tabbane
 Editions Hermès

Sens de la liaison	Montante	Descendante	
Partie réception	BTS	MS	
Sensibilité	- 104 dBm	- 102 dBm	C
Marge de protection (cf. 6.5.2.4)	3 dB	3 dB	D
Perte totale câble et connecteur	4 dB	0 dB	E
Gain d'antenne	12 dBi	0 dBi	F
Marge de masque (90 % de la surface)	5 dB	5 dB	G
Puissance médiane nécessaire	- 104 dBm	- 94 dBm	H=C+D+E-F+G
<hr/>			
Partie émission	MS	BTS	
Puissance d'émission (classes 2 et 7)	33 dBm	38 dBm	I
Perte de couplage + isolateurs	0 dB	3 dB	K
Perte totale câbles et connecteurs	0 dB	4 dB	L
Gain d'antenne	0 dBi	12 dBi	M
PIRE	33 dBm	43 dBm	N=I-K-L+M
<hr/>			
Bilan de liaison			
Affaiblissement maximal	137 dB	137 dB	O=N-H
Perte due au corps humain	3 dB	3 dB	P
Affaiblissement de parcours	134 dB	134 dB	O-P
Portée en extérieur	2 km		
Portée intérieure (marge de 15 dB)	0,7 km		

On considère un câble de perte 2 dB/100m d'une longueur de 120 mètres dans la station de base, un mobile de puissance 2W.

La portée est calculée sur la valeur O-P en considérant la loi d'Okumura Hata pour une zone urbaine (figure 6.4).

D'après [GSM 03.30 Annexe A.1].

Figure 6.6. Exemple de bilan de liaison pour GSM 900

➤ Exemple de système « HF » GSM-DCS

Bilan de liaison

Source : Réseaux GSM-DCS, p 149
 Lagrange, Godlewski, Tabbane
 Editions Hermès

Sens de la liaison	Montante	Descendante	
Partie réception	BTS	MS	
Sensibilité	-104 dBm	-102 dBm	C
Marge de protection (cf. 6.5.2.4)	3 dB	3 dB	D
Perte totale câble et connecteur	2 dB	0 dB	E
Gain d'antenne	18 dBi	0 dBi	F
Gain de diversité	5 dB	0 dB	F'
Marge de masque (90 % de la surface)	6 dB	6 dB	G
Puissance médiane nécessaire	-116 dBm	-91 dBm	H=C+D+E-F-F'+G
Partie émission	MS	BTS	
Puissance d'émission (classes 1 et 2)	30 dBm	42 dBm	I
Perte de couplage + isolateurs	0 dB	3 dB	K
Perte totale câbles et connecteurs	0 dB	2 dB	L
Gain d'antenne	0 dBi	18 dBi	M
PIRE	30 dBm	55 dBm	N=I-K-L+M
Bilan de liaison			
Affaiblissement maximal	146 dB	146 dB	O=N-H
Perte due au corps humain	3 dB	3 dB	P
Affaiblissement de parcours	143 dB	143 dB	O-P
Portée en extérieure		2 km	
Portée intérieure (marge de 15 dB)		0,7 km	

On considère un mobile de puissance 1W.

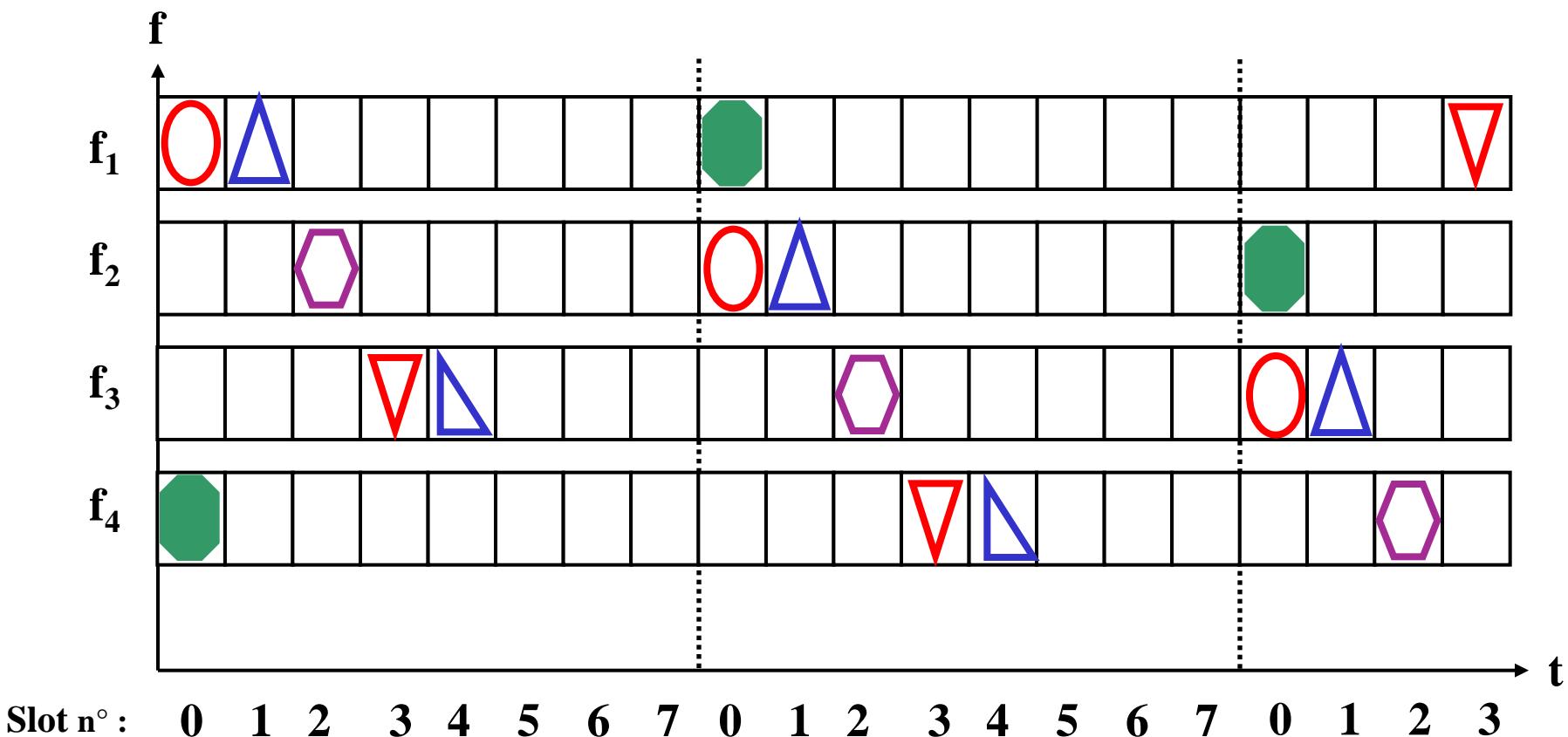
La portée est calculée en considérant la loi d'Hata pour une zone urbaine.

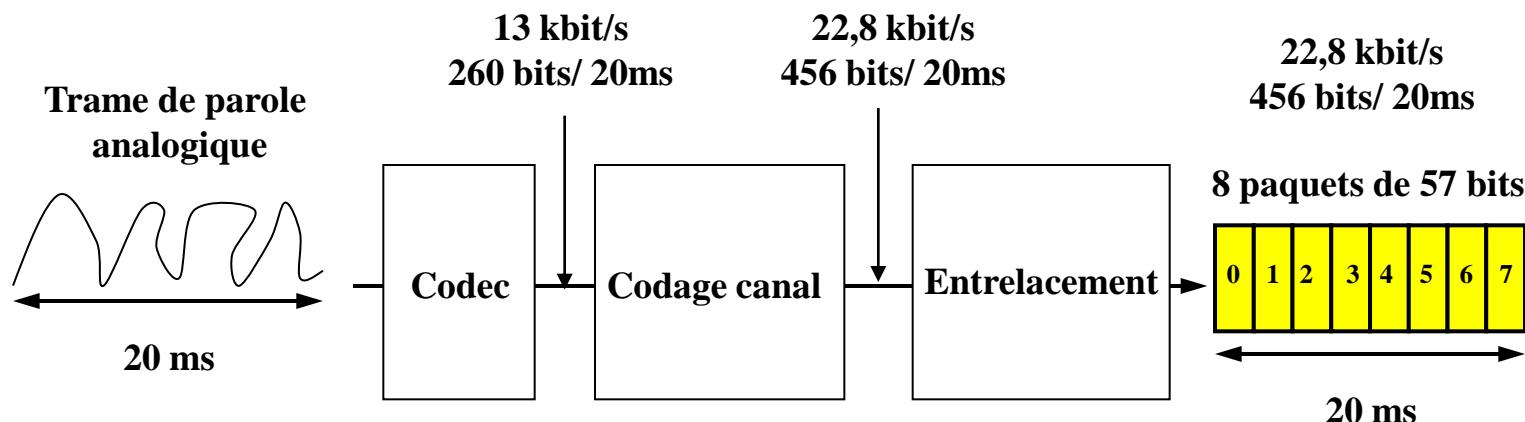
Noter l'utilisation de la diversité dans la station de base et l'utilisation d'antennes à fort gain pour supporter des affaiblissements plus importants que pour le bilan de liaison GSM 900 (ces techniques peuvent être aussi employées pour GSM 900).

D'après [GSM 03.30 Annexe A.3].

Figure 6.7. Exemple de bilan de liaison pour DCS 1800

Exemple de frequency hopping sur 4 fréquences





Séquence d'apprentissage

3	57	1	26	1	57	3	8,25
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Intervalle de garde

156,25 bits dont $2 \times 57 = 114$ bits de parole
Le débit passe à $(156,25/114) \times 22,8 \text{ kbit/s} = 31,25 \text{ kbit/s}$

Contenu d'un burst

